LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2017**

MT 3503 – VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 04-11-2017 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions **(10 x 2 = 20 marks)**

1. Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, -1).
2. Find 'a' such that $(3x-2y+z)\overline{i} + (4x+ay-z)\,\overline{j} + (x-y+2z)\overline{k}$ is solenoidal.
3. Define a conservative vector field.
4. If $\overline{A} = x^2\,\overline{i} + y^2\,\overline{j},$ evaluate $\int \overline{F}.\overline{dr}$ along the line $y = x$ from (0, 0) to (1, 1).
5. For any closed surface S, evaluate $\iint_S curl\,\overline{F}.\overline{ds}.$
6. State Green's theorem for a plane.
7. Solve: $4p^2 - 8p + 3 = 0, \text{where p=}\dfrac{dy}{dx}.$
8. Solve: $\dfrac{1}{x}\dfrac{dy}{dx} + \dfrac{y}{x}\tan x = \cos x.$
9. Solve: $(D^3 - 7D - 6)y = 0.$
10. Solve : $xy'' + y = 0.$

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. Find the directional derivative of ¤ $= (x^2 + y^2 + z^2)^{\frac{-1}{2}}$ at the point P (3, 1, 2) in the direction of the vector $yz\overline{i} + zx\overline{j} + xy\overline{k}$.
12. If $\overline{r} = x\,\overline{i} + y\,\overline{j} + z\,\overline{k}$ and r = $|\overline{r}|$, show that

    a) $\text{div}(\overline{\text{r}}¤) = 3¤ + \overline{r}.(grad¤)$

    $b)\,\text{div}(\hat{\text{r}}) = \dfrac{2}{r}.$ $(3+5)$
13. Evaluate $\iint_S \overline{\text{A}}.\hat{n}$ ds, where $\overline{\text{A}}\text{=z}\,\overline{\text{i}} + x\,\overline{j} - 3y^2 z\,\overline{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z =0 and z = 5.
14. Evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y = 0, x = \dfrac{®}{2}, y = \dfrac{2}{®}x.$
15. Solve: $x^2 = 1 + p^2.$

16. Solve: $\frac{dx}{dy}-\frac{2}{3}xy=x^4y^3.$

17. Solve: $y''+4y=4\tan 2x,$ using method of variation of parameters.

18. Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\left\{\log(1+x)\right\}.$

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. a) Show that the vector field $\overline{F}=\frac{\overline{r}}{r^3}$ is irrational as well as solenoidal. Find the scalar potential.

b) Evaluate $\iint_S \overline{A}.\hat{n}\,ds,$ where $\overline{A}=(x+y^2)\overline{i}-2x\overline{j}+2yz\overline{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant. **(10 +10)**

20. Verify divergence theorem for $\overline{F}=(x^2-yz)\overline{i}+(y^2-zx)\overline{j}+(z^2-xy)\overline{k}$ taken over the rectangular parallelepiped $0\le x\le a,\ 0\le y\le b,\ 0\le z\le c.$ **(20)**

21. a) Solve: $(1-x^2)y'+2xy=x\sqrt{1-x^2},$ given that y = 0 when x = 0.

b) Solve: $p^2+2yp\cot x-y^2=0.$

22. a) Solve: $x^3y'''+3x^2y''+xy'+y=x+\log x.$

b) Solve: $\frac{d^2y}{dx^2}-4y=x\sinh x.$

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