LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – NOVEMBER 2017

MT 5406- COMBINATORICS

Date13-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION-A

Answer all the questions. **(10×2=20)**

1. Find the number of 4-character words that can be formed using the 26 letters of the alphabet.
2. Find the number of ways of assigning 18 identical posters to 15 dormitories so that no dormitory receives more than 1 poster and the number of unassigned posters is a minimum.
3. Define the falling factorial polynomial
4. Find the coefficient of $x_1^2x_2^3x_3^4$in $(x_1+x_2+x_3)^9$.
5. Define Derangement and write the formula to find the number of derangements of *m* distinct objects D_m.
6. Find the sequences of the ordinary generating functions $2x^2(1-x)^{-1}$ and$(3+x)^3$.
7. Find the coefficient of x^{27} in $(x^4+x^5+x^6+\cdots)^5$ and in$(x^4+2x^5+3x^6+\cdots)^5$.
8. Write the exponential generating functions of the following functions:
   a) $\{1,2,3,0,0,0\ldots\}$
   b) $\{1,1,1,1,\ldots\}$
9. Calculate $per\begin{bmatrix}1 & 2\\ 1 & -1\end{bmatrix}$.
10. Evaluate $\phi(256)$.

SECTION-B

Answer any five questions. **(5×8=40)**

11. Define Stirling number of the second kind and derive its recurrence formula. Also construct the table of stirling numbers of the second kind when n=6 and m=6.
12. Prove that the number of distribution of *n* distinct objects into *m* distinct boxes with the order of objects in each box is significant and empty boxes are permitted is the rising factorial $[m]^n$.
13. Prove the multinomial theorem and hence find the coefficient of $x_1^2x_3x_4^3x_5^4$ in the expression$(x_1+x_2+x_3+x_4+x_5)^{10}$
14. Derive the formula to find the sum of first *n* natural numbers using its recurrence formula given by $a_n-a_{n-1}=n, n\geq 1$.
15. a) In an experiment, 4 differently colored dice are thrown simultaneously and the numbers are added. Find the numbers of distinct experiments such that 1) the total is 18 and 2) the total is 18 and the green die shows an even number.

    b) Find the number of ways of forming a committee of 9 people drawn from 3 different parties so that no party has an absolute majority in the party. (5+3)

16. Find the rook polynomial for the given chess board *C*:

17. Show that 97 is the twenty-fifth prime.

18. State and prove Burnside Frobenius theorem.

SECTION-C

Answer any two questions **(2×20=40)**

19. a) If m and n are positive integers, prove that the equation $x_1 + x_2 + \cdots + x_m = n$ has exactly $\frac{[m]^n}{n!}$ solutions in nonnegative integers x_k.

b) In a town council there are 10 democrates and 11 republicans. There are 4 women among the democrats and 3 women among the republicans. Find the number of planning committee of eight councellors such that of equal number of men and women and equal number of member of both committee. (12+8)

20. State and prove the Ménage problem.

21. a)Find the rook polynomial for a 2×2 Chess board by the use of expansion formula.

b) An executive attending a week long seminar has 5 suits of different colors. On Mondays she does not wear blue or green, on Tuesdays she does not wear red or green, on Wednesdays she does not wear blue or whiteor yellow, on Fridays, she does not wear white. How many ways can she does without repeating a color for the seminar?

(6+14)

22. Find all elements of the group *G* of symmetries of a square. Count the distinguishable colorings of the 4 vertices, if each vertex is to be either red or blue. Exhibit in a diagram the patterns. Also find the Pattern Inventory of *G*.

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