LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2018**

16UMT5MC01– REAL ANALYSIS

Date: 27-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions. (10 × 2 = 20)

1. When do you say that a set is order complete?
2. If a and b are real numbers show that $|a+b| \geq \left||a| - |b|\right|$.
3. If (M,d) is a discrete metric space ,find B(x; 1).
4. Define compact set and give an example of it.
5. Show that every convergent sequence is a Cauchy sequence in R.
6. When do you say that a function f:X→Y is continuous at x in X?
7. Show that every differentiable function is also continuous.
8. Define local minimum and local maximum of a function at a point.
9. Define a strictly increasing function and strictly decreasing function.
10. When do you say that a function f is of bounded variation on [a,b]?

PART – B

Answer any FIVE questions (5 × 8 = 40)

11. State and prove Archimedean property.
12. Show that every subset of a countable set is countable.
13. Let Y be a subspace of a metric space X. Show that a subset A of Y is open in Y if and only if A = Y ∩ G for some set G open in X.
14. Prove that a closed subset of a compact metric space is compact.

15. Let X,Y be metric spaces, S be a non empty subset of X, f:X→Y and x_0 be an accumulation point of S. Show that $\lim_{x \to x_0} f(x)$ = y_0 if and only if for every sequence $\{x_n\}$ of points in $S - \{x_0\}$, that converges to x_o, $\{f(x_n)\}$ converges to y_0 .

16. Let f(x) = x^2 for x in R. Show that f is continuous but not uniformly continuous.

17. State and prove Rolle's theorem.

18. If f is monotonic on [a,b] , show that the set of all discontinuities of f is countable.

PART – C

Answer any TWO questions. **(2 × 20 =40)**

19. (a) Show that e = 1+ $\frac{1}{1!}$ + $\frac{1}{2!}$ + ... is irrational.

(b) State and prove Cauchy-Schwarz inequality.

20. (a) Let S be a subset of $\mathbf{R^n}$. If every infinite subset of S has an accumulation point in S, show that S is closed and bounded.

(b)Show that every compact subset of a metric space is complete.

21. (a) State and prove Taylor's theorem.

(b) State and prove intermediate value theorem for derivatives.

22. (a) State and prove Chain rule for differentiation.

(b) Let f be a bounded variation on [a, b] and c ∈ (a, b), then prove that f is bounded variation on [a, b] as well as [c, b] and V_f[a, b] = V_f[a, c] + V_f[c, b].

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