LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2018**

16UMT5MC03– LINEAR ALGEBRA

Date: 01-11-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

ANSWER ALL THE QUESTIONS $(\mathbf{10 * 2 = 20}\boldsymbol{marks})$

1. Define a vector space over a field F.
2. Determine whether $T: R^2 \to R^3$ defined by $T(a,b) = (a+1, 2b, a+b)$ is a vector space homomorphism or not.
3. Define inner product space.
4. Let R^3 be the inner product space over R under the standard inner product. Find the norm of (3,0,4).
5. If $T \in A(V)$ and $\lambda \in F$ and λ is a characteristic root of T then prove that $\lambda I - T$ is singular.
6. Define singular and regular linear transformation.
7. Define matrix of a linear transform.
8. Define similar matrices.
9. Define unitary linear transformation.
10. If $T \in A(V)$ is Hermitian, then prove that all its characteristic roots are real.

PART B

ANSWER ANY FIVE QUESTIONS $(\mathbf{5 * 8 = 40}\boldsymbol{marks})$

11. If V is a vector space over F then show that

    i) $\alpha 0 = 0$ for $\alpha \in F$

    ii) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F, v \in V$.

    iii) If $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.
12. If $v_1, \ldots, v_n$ is a basis of V over F and if $w_1, \ldots, w_m$ in V are linearly independent over F then prove that $m \leq n$.
13. State and prove Schwartz inequality.
14. If V is finite-dimensional over F then prove that $T \in A(V)$ is singular if and only if there exists a $v \neq 0$ in V such that $vT = 0$.

15. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

16. If V is $n-$dimensional over F and if $T \in A(V)$ has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F.

17. If $T \in A(V)$ then prove that $T^* \in A(V)$, also prove that

i) $(T^*)^* = T$;

ii) $(S+T)^* = S^* + T^*$;

iii) $(\lambda S)^* = \bar{\lambda} S^*$;

iv) $(ST)^* = T^* S^*$;

for all $S, T \in A(V)$ and all $\lambda \in F$.

18. If λ is a characteristic root of the normal transformation N and if $vN = \lambda v$,then prove that $vN^* = \bar{\lambda} v$.

PART C

ANSWER ANY TWO QUESTIONS $(\mathbf{2 * 20 = 40})$

19. a) If V is finite-dimensional and if W is a subspace of V, then prove that $W$is finite-dimensional, $dimW \leq dimV$ and $\dim {}^{V}/_{W} = \dim V - \dim W$.

b) If V and W are of dimensions $m$and$n$, respectively, over F, then prove that $Hom(V, W)$ is of dimension mn over F. **(10+10)**

20. a) State and prove Gram-Schmidt orthonormalization process.

b) If Va finite-dimensional inner product space is and if Wa subspace of is V then prove that $V = W + W^{\perp}$.**(15+5)**

21. a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if maps V onto V.

b) If $\lambda_1, \lambda_{2,} \ldots \lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_{2,} \ldots v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_{2,} \ldots \lambda_k$, respectively then prove that $v_1, v_{2,} \ldots v_k$are linearly independent over F. **(10+10)**

22. a) If V is n-dimensional over F, and if $T \in A(V)$has the matrix $m_1(T)$ in the basis $v_1, \ldots v_n$and the matrix $m_2(T)$ in the basisand $w_1, \ldots w_n$of Vover F, then prove that there is an element C in F_n such that $m_2(T) = Cm_1(T)C^{-1}$.

b) If $V$is finite-dimensional over F, then for$T, S \in A(V)$ prove that

i) $r(ST) \leq r(T)$

ii) $r(TS) \leq r(T)$

iii) $r(ST) = r(TS) = r(T)$ for$S$ is regular in $A(V)$. **(10+10)**

****