LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2019**

16UMT6MC03 – DISCRETE MATHEMATICS

Date: 01-11-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part A (Answer ALL questions) **(10X2 = 20)**

1. Construct the truth table for $\neg P \wedge Q$.
2. What is the dual of $(P \vee Q) \wedge R$?
3. Write down the max terms of P and Q.
4. Define disjunctive normal form.
5. Define semigroup homomorphism.
6. Define monoid and give an example.
7. Define lattice.
8. State distributive inequality of lattice.
9. Define Boolean algebra.
10. State De Morgan's law for Boolean algebra.

Part B (Answer any FIVE questions) **(5 x 8 = 40)**

11. Construct the truth table for $\neg(P \wedge Q)$€ $(\neg P \vee \neg Q)$.
12. Write the following statements into symbolic form.
    (i) Mark is poor but happy.
    (ii) Mark is rich or sad.
    (iii) Mark is neither rich nor sad.
    (iv) Mark is poor or Mark is both rich and unhappy.
13. Determine the conjunctive normal form of $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$.
14. Show that $\neg P$ follows logically from the premises $\neg(P \wedge \neg Q)$, $(\neg Q \vee R)$, $\neg R$.
15. Prove that every cyclic monoid is commutative.
16. Let $(S,*)$ be a semigroup and R be a congruence relation on $(S,*)$. Then prove that the quotient group S/R is a semigroup $(S/R,\oplus)$, where the operation $\oplus$ corresponds to the operation * on S.
17. State and prove any two properties of lattice.

18. Define (i) complete lattice (ii) complemented lattice (iii) distributive lattice (iv) bounded lattice.

Part C (Answer any TWO questions) **(2 x 20 = 40)**

19. a) Construct the truth table for $(P \to Q) \wedge (Q \to P)$.

b) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

(8 + 12)

20. a) Obtain the principal disjunctive normal form of $(\neg P \to R) \wedge (Q \text{€} \quad P)$.

b) Prove that the composition of semigroup homomorphism is also a semigroup homomorphism.

(10 + 10)

21. a) State and prove isotonic property in lattice.

b) Define the following:

(i) Lattice homomorphism (ii) Direct product of two lattices.

(12 + 8)

22. a) Prove that the complement is unique in a complemented distributive lattice.

b) Write down the following Boolean expression in an equivalent sum of the products canonical form in three variables x_1, x_2, x_3.

(i) $x_1 * x_2$ (ii) $x_1 \oplus x_2$ (iii) $(x_1 \oplus x_2)' * x_3$.

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