LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

18PMT3ES02 – DIFFERENTIAL GEOMETRY

Date: 06-11-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Answer all the questions

1. (a) Obtain the equation of the tangent at a point on the curve of intersection of two surfaces $f_1(x,y,z) = 0$ and $f_2(x,y,z) = 0$. (5)

(OR)

(b) Find the centre and radius of the osculating circle. (5)

(c) Derive the equation of the osculating plane at the point on the space curve and hence find the equation of the osculating plane for the vector $\vec{r} = (u, u^2, u^3)$. (15)

(OR)

(d) State and prove Serret Frenet formulae. (15)

2. (a) Show that if the circle $lx + my + nz = 0, x^2 + y^2 + z^2 = 2cz$ has three point of contact at the origin with the paraboloid $ax^2 + by^2 = 2z$ then $c = \frac{l^2+m^2}{bl^2+am^2}$. (5)

(OR)

(b) Define (i) inflexinol tangent (ii) involute and evolute of a space curve (iii) helix. (5)

(c) Show that the intrinsic equation of the curve $x = ae^u \cos u, y = ae^u \sin u, z = be^u$ are $k = \frac{a\sqrt{2}}{s\sqrt{2a^2+b^2}}$ and $\tau = \frac{b}{s\sqrt{2a^2+b^2}}$. (15)

(OR)

(d) State and prove fundamental theorem of space curves. (15)

3. (a) What are the types of singularities? Explain briefly. (5)

(OR)

(b) Find the equation for the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. (5)

(c) Explain the first fundamental form of a surface and give its geometrical interpretation. (15)

(OR)

(d) (i) Derive the equation of polar developable associated with a surface.

(ii) Write a brief note on tangent plane and normal plane. (8+7)

4. (a) State and prove Meusnier's theorem. (5)

(OR)

(b) Define oblique section, normal section, principal section and line of curvature. (5)

(c) (i) Explain the different points on a surface in detail.
(ii) State and prove Euler's theorem. (7+8)

(OR)

(d) Derive the equation satisfying principal curvature at a point on a surface and the equation of principal direction at a point. (15)

5. (a) Derive Weingarton equation. (5)

(OR)

(b) Define umbilic point and state the partial differential equation of surface theory. (5)

(c) State the fundamental theorem of Surface Theory and demonstrate it in the case of unit sphere. (15)

(OR)

(d) Derive Mainardi Codazzi equation. (15)

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