LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2019**

MT 1503 – ANALYTICAL GEOMETRY OF 2D,TRIG. & MATRICES

Date: 01-11-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Part A (Answer ALL questions) **(10x2 = 20)**

1. Write the expansion of $\tan 3\theta$ in powers of $\tan\theta$.
2. Expand $\cos\theta$ in a series of powers of θ.
3. Show that $\cosh^2 x + \sinh^2 x = \cosh 2x$.
4. Find $Log(1-i)$.
5. Determine the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.
6. State Cayley-Hamilton theorem.
7. What is the pole of the line $2x+3y+4=0$ with respect to the parabola $y^2 = 4ax$?
8. Define conjugate of two diameters of an ellipse.
9. What is the formula to find the distance between the points $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$?
10. Define rectangular hyperbola.

Part B (Answer any FIVE questions) **(5 x 8 = 40)**

11. Expand $\sin^7\theta$ in a series of sines of multiples of θ.
12. Evaluate $\lim_{\theta \to 0} \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$.
13. If $\sin(A+iB) = x+iy$, prove that (i) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$ (ii) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$.
14. Determine the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find its inverse using Cayley-Hamilton theorem.
15. Compute the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
16. Find the locus of the poles of all tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$.

17. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

18. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

Part C (Answer any TWO questions) **(2 x 20 = 40)**

19. a) Prove that $\frac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

b) Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiples of θ.

(10 + 10)

20. a) Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

b) Deduce the expansion of $\tan^{-1}x$ in powers of x from the expansion of $\log(a+ib)$.

(12 + 8)

21. Diagonalise the matrix $A = \begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$.

(20)

22. a) A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose centre is C meets the circle $x^2 + y^2 = a^2 + b^2$ at Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.

b) Trace the curve $\frac{12}{r} = 4 + \sqrt{3}\cos\theta + 3\sin\theta$. (12 + 8)

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