LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2019**

MT 5505 – REAL ANALYSIS

Date: 29-10-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Part - A

Answer ALL questions: **(10 x 2 = 20)**

1. Define upper bound. Give an example.
2. Define a countable set.
3. Define a metric space.
4. Define an open set.
5. Prove that every convergent sequence is Cauchy.
6. Define continuity at a point.
7. State generalized mean value theorem.
8. Define local maximum of a function at a point.
9. Define a function of bounded variation.
10. Define Riemann Stieltjes integral.

Part - B

Answer any FIVE questions: **(5 x 8 = 40)**

11. State and prove Minkowski's inequality.
12. Prove that every subset of a countable set is countable.
13. State and prove Bolzano theorem.
14. Prove that the Euclidean space is complete.
15. Define convergent sequence and prove that a sequence cannot converge to two distinct limits.
16. State and prove Rolle's theorem.
17. State and prove intermediate value theorem for derivatives.
18. If f is monotonic on $[a,b]$, then prove that the set of all discontinuities of f is countable.

Part - C

Answer any TWO questions: **(2 x 20 = 40)**

19. (a) State and prove Cauchy - Schwarz inequality.
    (b) Prove that $\boldsymbol{R}$ is uncountable. **(10+10)**
20. (a) Let $\boldsymbol{Y}$ be a subspace of a metric space $(\boldsymbol{X}, d)$. Then prove that a subset $\boldsymbol{A}$ of $\boldsymbol{Y}$ is open in $\boldsymbol{Y}$ if and only if $\boldsymbol{A}=\boldsymbol{Y} \cap \boldsymbol{G}$ for some set $\boldsymbol{G}$ open in $\boldsymbol{X}$.
    (b) Prove that the continuous image of a compact metric space is compact. **(10+10)**
21. State and prove Taylor's theorem. **(20)**
22. (a) State and prove integration by parts.
    (b) State and prove the linear property of Riemann Stieltjes integral. **(10+10)**

****