

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2019

UMT 1303 – MATHEMATICS FOR STATISTICS

Date: 05-11-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer ALL the questions:

(10 X 2 = 20)

1. If $y = 4x^3 - 2x + \frac{3}{x^3}$, find $\frac{dy}{dx}$.
2. Differentiate e^t with regard to \sqrt{t} .
3. Prove that the function $f(x) = x^3 - 3x^2 + 6$ is positive for all value of $x \geq 2$.
4. For what values of x is the curve $y = 3x^2 - 2x^3$ concave upwards and when it is convex upwards?
5. Find the first order partial differential coefficients of $u = \log(ax + by + cz)$.
6. State Euler's theorem.
7. Evaluate $\int \frac{dx}{\sqrt{4-9x^2}}$.
8. Integrate $\frac{\sin(\log x)}{x}$ with respect to x .
9. Prove that $\int_0^{\frac{\pi}{2}} \cos^2(x) dx = \int_0^{\frac{\pi}{2}} \sin^2(x) dx$.
10. Find $\int_0^{\frac{\pi}{2}} \sin^6 x dx$.

PART B

Answer any FIVE questions:

(5 X 8 = 40)

11. If $y = \frac{2x e^x \sin x}{(1+x^2)^2(1-x^2)^2}$, find $\frac{dy}{dx}$.
12. Show that for $x > 0$, $x - \frac{1}{2}x^2 < \log(1+x) < x$.
13. Find the n^{th} differential coefficient of $x^3 \log 2x$.
14. If $u = \log(x^2 + y^2 + z^2)$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2 + y^2 + z^2}$.
15. Verify Euler's theorem for the function $u = x^3 + y^3 + z^3 + 3xyz$.
16. Evaluate $\int \frac{6x+5}{\sqrt{6+x-2x^2}} dx$.
17. Derive the reduction formula for $I_n = \int x^n \cos ax dx$ and evaluate $\int x^2 \cos 3x dx$.
18. Prove that $\int_0^{\pi} \log(1 + \cos x) dx = \pi \log\left(\frac{1}{2}\right)$.

PART C

Answer any TWO questions:

(2 X 20 = 40)

19. (a) If $(\sin x)^{\cos y} = (\sin y)^{\cos x}$, find $\frac{dy}{dx}$.
(b) Differentiate $\log \left(\frac{x^2+x+1}{x^2-x+1} \right)$ with respect to $(\sin x)^{x \cos x}$.
(c) Find y_n , where $y = \frac{3}{(x+1)(2x-1)}$.
(5+7+8)
20. (a) Find the points of inflexion on the cubic $y = \frac{a^2x}{x^2+a^2}$ and show that they lie on a straight line.
(b) If $u = \frac{xy}{x+y}$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
(12+8)
21. (a) Find the maximum or minimum values of the function $u = 2(x^2 - y^2) - x^4 + y^4$.
(b) Evaluate $\int \frac{2x+3}{x^2+x+1} dx$.
(12+8)
22. (a) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$.
(b) Integrate $x^3 e^{2x}$ with respect to x .
(12+8)
