

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2019

UMT 1501 – ALGEBRA

Date: 30-10-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions.

(10 X 2 = 20)

1. Form a quadratic equation, given that $-2 + \sqrt{-7}$ is a root.
2. Solve the equation $x^3 + 6x + 20 = 0$, one root being $1 + 3i$.
3. Find the number of real roots of the equation $x^3 + 18x - 6 = 0$.
4. Find the interval in which a root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ lies.
5. State Cayley Hamilton theorem.
6. Find $\frac{e+e^{-1}}{2}$ and $\frac{e-e^{-1}}{2}$.
7. Find the characteristic equation of the matrix $\begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$.
8. Define similar matrices.
9. Find the number of integers less than and prime to 720.
10. Use Binomial theorem to find the seventh power of 11.

PART – B

Answer any FIVE questions:

(5 X 8 = 40)

11. Find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$, where α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$
12. Diminish the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and write the transformed equation.
13. State and prove Fermat's theorem.
14. Find the sum to infinity of the series $1 + \frac{2}{6} + \frac{2 \cdot 5}{6 \cdot 12} + \frac{2 \cdot 5 \cdot 8}{6 \cdot 12 \cdot 18} + \dots$
15. Find the sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \frac{1+2+2+2^3}{4!} \dots$

16. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find its inverse.
17. Verify Cayley Hamilton theorem for the matrix $= \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
18. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

PART- C

Answer Any TWO Questions.

(2 X 20 = 40)

19. a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression
 b) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ **(8 + 12)**
20. a) Calculate the root of the equation $x^3 - 3x + 1 = 0$ to two places of decimal which lies between 1 and 2 by using Horner's method.
 b) Solve the equation $x^3 - 6x - 9 = 0$ using Cardon's method. **(10 + 10)**
21. a). Show that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots$
 b). State Wilson's theorem and prove that $(18)! + 1$ is divisible by 437. **(10 + 10)**
22. Diagonalize the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. **(20)**
