LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

PMT1MC02 – REAL ANALYSIS-I

Date: 25-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**Answer the following (5 x 1 = 5 Marks)**		
a)	Give an example of an open set.	K1	CO1
b)	Define derivates of higher order.	K1	CO1
c)	Define Refinement of partition of a set.	K1	CO1
d)	State any two properties of definite integral.	K1	CO1
e)	Give an example of a function which is continuous but not uniformly continuous.	K1	CO1
2.	**Choose the correct answer for the following (5 x 1 = 5 Marks)**		
a)	The set of all complex numbers in R^2 is (i) closed (ii) open (iii) perfect (iv) bounded	K2	CO1
b)	The set of points where $f(x) = \frac{x}{1+\|x\|}$ is differentiable at (i) $(-\infty, 1) \cup (1, \infty)$ (ii) $(-\infty, \infty)$ (iii) $(0, \infty)$ (iv) $(-\infty, 0)$	K2	CO1
c)	Let $f(x) = x, 0 \leq x \leq 3$ and let $P = \{0,1,2,3\}$ be a partition of $[0,3]$, then the value of $U(P, f)$ is (i)1 (ii)2 (iii) 3 (iv) 6	K2	CO1
d)	If $f(x) = x, 0 \leq x \leq 1$ then the value of $\int_0^1 x dx$, where $f \epsilon R[0,1]$ is (i)0 (ii) $\frac{1}{2}$ (iii) 1 (iv) $-\frac{1}{2}$	K2	CO1
e)	$\lim_{n\to\infty} 3 + (-1)^n$ is (i) divergent (ii) convergent (iii) oscillatory (iv) harmonic	K2	CO1
SECTION B			
Answer any THREE of the following: (3 x 10 = 30 Marks)			
3.	a) Are compact subsets of metric space closed? If yes prove it. b) Prove that a set E is open if and only if its complement is closed. **(5+5)**	K3	CO2
4.	Suppose f is continuous on $[a, b]$, $f^{-1}(x)$ exists at some point $x \epsilon [a, b]$ is defined on the interval I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$ then h is differentiable at x and $h'(x) = g'(f(x))f'(x)$.	K3	CO2
5.	If $f \epsilon \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.	K3	CO2
6.	Is limit of the integral equal to integral of the limit for i) $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ ii)	K3	CO2

	$f_n(x) = n^2x(1-x^2)^n$?		
7.	If P^* is a refinement of P, then prove that $L(P,f,\alpha) \leq L(P^*,f,\alpha)$ and $U(P^*,f,\alpha) \leq U(P,f,\alpha)$.	K3	CO2

SECTION C

Answer any TWO of the following: (2 x 12.5 = 25 Marks)

8.	Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be e limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n$, then prove that $\{A_n\}$ converges and $\lim_{t\to x} f_n(t) = \lim_{n\to\infty} A_n$ implies $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{n\to\infty} \lim_{t\to x} f_n(t)$.	K4	CO3
9.	Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\epsilon > 0$ there exists an integer N such that $m \geq N, n \geq N$, $x \epsilon E$ implies that $\lvert f_n(x) - f_m(x)\rvert \leq \epsilon$.	K4	CO3
10.	Briefly explain existence of best approximation.	K4	CO3
11.	Assume α increasing monotonically and $\alpha' \epsilon \mathcal{R}$ on $[a,b]$, let f be a bounded real function on $[a,b]$. Then prove that $f \epsilon \mathcal{R}$ if and only if $f\alpha' \epsilon \mathcal{R}$ and $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)\, dx$.	K4	CO3

SECTION D

Answer any ONE of the following: (1 x 15 = 15 Marks)

12.	a)	If f is a continuous mapping of a compact metric space X into a metric space Y then, prove that f is uniformly continuous on X.	K5	CO4
	b)	Develop and justify sets for the following conditions i) closed, open, perfect, and not bounded. ii) closed, not open, perfect, and bounded. iii) closed, not open, not perfect and bounded. iv) closed, not open, not perfect, and not bounded. v) closed, open, perfect, and not bounded. (10+5)	K5	CO4
13.	a)	Prove the integral formula of integration by parts using Reimann-Stieltjes integral.	K5	CO4
	b)	Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. (7+8)	K5	CO4

SECTION E

Answer any ONE of the following (1 x 20 = 20 Marks)

14.	Discuss whether a uniformly continuous polynomial P_n is real for a continuous complex function f in $[a,b]$.	K6	CO5
15.	What is the condition for Lagrange's mean value theorem and Rolle's theorem from generalized mean value theorem? Discuss it.	K6	CO5

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