LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

PMT1MC06 – PROBABILITY THEORY AND RANDOM PROCESSES

Date: 02-12-2022 Dept. No. ☐ Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**(5 x 1 = 5)**		
a)	State Bernoulli's law of large numbers.	K1	CO1
b)	Identify the relation between the regression coefficients and correlation coefficient.	K1	CO1
c)	Tell the conditions for minimum variance unbiased estimators.	K1	CO1
d)	Define power of the test in statistical hypothesis.	K1	CO1
e)	Define Markov chain.	K1	CO1
2.	**(5 x 1 = 5)**		
a)	If X and Y are independent, then identify which of the following is true? (i) $E(XY) = E(X)E(Y)$ (ii) $E(XY) \leq E(X)E(Y)$ (iii) $E(XY) \geq E(X)E(Y)$ (iv) None	K2	CO1
b)	Two variables x and y are related $y = ax + b$, where a and b are constants, then the value of γ is (i) +1 (ii) -1 (iii) ± 1 (iv) 0	K2	CO1
c)	If T is an unbiased estimator of θ, then T^2 is identified as (i) Biased estimator of θ^2 (ii) Unbiased estimator of θ^2 (iii) Normal estimator of θ^2 (iv) Binomial estimator of θ^2.	K2	CO1
d)	The level of significance is estimated using the probability of (i) Type I error (ii) Type II error (iii) accepting H_0 when H_0 is false (iv) None	K2	CO1
e)	The mean of the random process is often expressed as (i) Rank correlation (ii) Variance (iii) ensemble average (iv) auto-correlation	K2	CO1

SECTION B

Answer any THREE of the following **(3 x 10 = 30)**

3. State and prove weak law of large numbers and hence examine whether the law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as follows: $P[X_k = \pm 2^k] = 2^{-(2k+1)}$; $P[X_k = 0] = 1 - 2^{-2k}$. K3 CO2

4. Calculate the correlation coefficient between the weight of Mother and daughter from the following data: K3 CO2

Weight of Mother (in Kgs.)	64	65	66	67	68	69	70
Weight of Daughter (in Kgs.)	66	67	65	68	70	68	72

5. A random sample (X_1, X_2, X_3) of size 3 is drawn from a normal population with mean μ and variance σ^2. Let T_1, T_2 and T_3 are the estimators used to estimate mean value μ, where $T_1 = X_1 + X_2 - X_3, T_2 = 2X_1 + 3X_3 - 4X_2$ and $T_3 = \frac{1}{3}(\beta X_1 + X_2 + X_3)$. K3 CO2

(i) Are T_1 and T_2 unbiased estimators?

(ii) Calculate the value of β such that T_3 is unbiased estimator for μ.

Determine the best estimator.

6. If $x \geq 1$ is the critical region for testing $H_0: \theta = 2$ against the alternative $H_1: \theta = 1$, on the basis of the single observation from the population, $f(x, \theta) = \theta \exp(-\theta x), 0 \leq x < \infty$. Obtain the values of type I and type II errors. Also, calculate the level of significance and power of the test. K3 CO2

7. Suppose a communication system transmits the digits 0 and 1 through many stages. At each stage the probability that the same digit will be received by the next stage as transmitted is 0.75. Using Chapman-Kolmogorov equations, calculate the probability of receiving 0 in the 5th stage by entering 0 at the first stage.

SECTION C

Answer any TWO of the following **(2 x 12.5 = 25)**

8. State and prove Khintchin's theorem. In addition, discuss its application through an example. K4 CO3

9. (a) State and prove the invariance property of consistent estimators. **(5 marks)** K4 CO3

(b) Compare the marks obtained by eight students in the papers of Physics and Mathematics by using rank correlation method. K4 CO3

Marks in Physics	15	20	27	13	45	60	20	75
Marks in Maths	50	30	55	30	25	10	30	70

(7.5 marks)

10.	If T_1 and T_2 are minimum variance unbiased estimators for $\gamma(\theta)$, then prove that $T_1 = T_2$, almost surely.	K4	CO3
11.	A class of modulated signals is modelled by the process $Y(t) = AX(t)\cos(\omega t + \theta)$, where $X(t)$ is the message signal which is a random process with mean 0 and autocorrelation function $R_{XX}(\tau)$. $A\cos(\omega t + \theta)$ is the carrier with amplitude A and the frequency ω are constants and the initial carrier phase θ is uniformly distributed in $[-\pi, \pi]$. Also $X(t)$ and θ are independent. Show that $Y(t)$ is a WSS process.	K4	CO3

SECTION D

Answer any ONE of the following (1 x 15 = 15)

<table>
<tr><td>12.</td><td>State and prove Neyman-Pearson lemma.</td><td>K5</td><td>CO4</td></tr>
<tr><td>13.</td><td>A panel of judges A and B graded seven debtors and independently awarded the following marks:
<table>
<tr><td>Debtors</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>Marks of A</td><td>40</td><td>34</td><td>28</td><td>30</td><td>44</td><td>38</td><td>31</td></tr>
<tr><td>Marks of B</td><td>32</td><td>39</td><td>26</td><td>30</td><td>38</td><td>34</td><td>28</td></tr>
</table>
An eight debtor was awarded 36 marks by judge A while judge B was not present. If judge B were also present, how many marks would you except him to be awarded to the eighth debtor assuming that the same degree of relationship exists in their judgements?</td><td>K5</td><td>CO4</td></tr>
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SECTION E

Answer any ONE of the following (1 x 20 = 20)

14.	(a)	A discrete variate X takes the value x with probability $2^{-x}, x = 1, 2, ...$ Prove that Chebyshev's inequality gives $P(\lvert X - 2\rvert \geq 2) \leq \frac{1}{2}$. Also, determine its actual probability. **(8 marks)**	K6	CO5
	(b)	Explain the development of random process in the application of telegraph signal process. **(12 marks)**		
15.		Construct a dataset related to physical science and explain how regression lines can be used to predict an unknown data.	K6	CO5

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