# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

**PMT 3601 – COMBINATORICS**

Date: 02-12-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

**PART – A**

**Answer all questions** **(5 x 20 = 100 Marks)**

**Marks**

1 (a) Consider the letters of the word DELHI. How many words of length 1,2,3,4,5 can be formed whether or not meaningful, using the letters. (5)
a. repetition of letters allowed.
b. repetition is not allowed.

(or)

(b) There are 16 books on a bookshelf. In how many ways can 6 of these books be selected if a selection must not include two neighboring books? (5)

(c) Derive the Stirling numbers of the first and second kind and tabulate the value for $S_7^7$ (15)

(or)

(d) Consider the letters of the word UNIVERSAL. How many words of length 1, 2, 3, 4, 5, 6, 7, 8, 9 can be formed whether or not meaningful, using the letters. (15)
a. repetition of letters allowed.
b. repetition is not allowed.

2 (a) A child has a store of toy letters consisting of a 4 $A's$, 3 $B's$ and 2 $E's$. (5)
i.) How many different increasing four- letter words (given $A < B < E$) can it make? The child does not worry about the meaning of the words.

ii.) Show that there exists a bijection between the set of increasing four – letter words of all possible lengths and the set of terms of the expansion $(1 + A + A^2 + A^3 + A^4)(1 + B + B^2 + B^3)(1 + E + E^2)$. (or)

(b) Verify that the number of increasing words of length 10 out of the alphabet {a, b, c, d, e} with $a < b < c < d < e$ is the coefficient of $t^{10}$ in $(1 - t)^{-5}$. Try to explain why this is so. (5)

(c) A family of 3, another family of 2, and two bachelors go for a joy ride in a giant wheel in which there are three swings $A, B, C$. In how many ways can they be seated in the swings (assuming there are sufficient number of seats in each swing) if the families are to be together? List all the ways. (15)

(or)

(d) Briefly explain the difference between ordinary generating function and the exponential generating function with an example. (15)

3 (a) Obtain determinantal expression for $s_r$ in terms of (5)
a. the $a_r{}'s$ and
b. the $h_r{}'s$

(or)

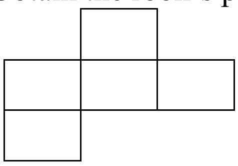
(b) Find the coefficient of $\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4$ in $h_4 h_3$ by computing $\phi_{43,2221}$. (5)

(c) Briefly explain the four types of symmetric functions. (15)

(or)

(d) Given $\lambda \vdash N$ prove that $k_\lambda$ is a linear combination of the $s_\mu$'s. (15)

4 (a) Obtain the rook's polynomial for the following chess board. (5)



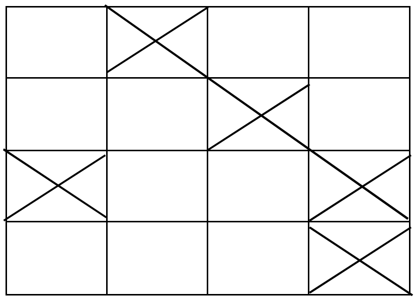
(or)

(b) Seven people enter a lift. The lift stops only at three floors (unspecified). At each of these three floors, no one enters the lift, but at least 1 person leaves the lift. After the three floor stops, the lift is empty. In how many ways can this happen? (5)

(c) Briefly explain the problem of Fibonacci with an example (15)

(or)

(d) If a chess board has $4 \times 4$ boxes in it with following forbidden positions (15)



Find the rook's polynomial.

5 (a) The defense of a country decides on a code of three digits selected from the Indo – Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. But the slip on which the code is hand – written allows confusion between the top and bottom. In other words, the top and bottom are indistinguishable. So a code such as 9180 cannot be distinguished form 0816. How many distinguishable codes are there? (5)

(or)

(b) How many distinct circular necklace patterns are possible with 6 beads, these beads being available in three different colors? Can you make an inventory of them? (5)

(c) Find the cycle structure of the permutations on the set of 8 vertices, 6 faces and 12 edges induced by the following category of rotations of the cube (15)

a. $90^o$ rotations about the axes joining the centers of the opposite faces. There are six in all.

b. $180^o$ rotations about each of the axes mentioned in a. there are three.

c. $120^o$ rotations about the axes joining the opposite edges. There are six in all.

d. $180^o$ rotations about the axes joining the midpoints of the opposite edges. There are six in all.

e. The identity.

(or)

(d) State and prove Polya's enumeration theorem. (15)

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