LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

PMT 3602 – DIFFERENTIAL GEOMETRY

Date: 02-12-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Answer ALL the questions

1. (a) Prove that the curvature is the rate of change of the angle of contingency with respect to the arc length. (5)

(OR)

(b) Find the length of the circular helix $\vec{r} = a\cos u\,\vec{i} + a\sin u\,\vec{j} + bu\,\vec{k}$, $-\infty < u < \infty$ varies from the point $(a, 0,0)$ to $(a, 0,2\pi b)$. Also obtain the equation in terms of parameter s. (5)

(c) Define an osculating plane and derive the equation of the osculating plane at the point on the space curve. (15)

(OR)

(d) State and prove Serret-Frenet formulae. (15)

2. (a) Find the plane that has three points of contact at origin with the curve $x = u^4 - 1$, $y = u^3 - 1, z = u^2 - 1$. (5)

(OR)

(b) Prove that the necessary and sufficient condition that a curve be of constant slope is that the ratio of curvature to the torsion is a constant. (5)

(c) Derive Riccati equation. (15)

(OR)

(d) Derive the equation of the curvature and torsion of the evolute of a curve. (15)

3. (a) What are the types of singularities? Explain briefly. (5)

(OR)

(b) Write a brief note on tangent plane and normal plane. (5)

(c) Explain the first fundamental form of a surface and give its geometrical interpretation. (15)

(OR)

(d) Derive the equation of rectifying developable and tangential developable associated with a surface. (15)

4. (a) Prove that the value of the second fundamental form at any point P is equal to twice the length of the perpendicular from the neighbouring point Q on the tangent plane at P.

(5)

(OR)

(b) With usual notations, prove that the necessary and sufficient condition that the lines of curvature may be a parametric curve is that $f = 0$ and $F = 0$. (5)

(c) Find the first and second fundamental form of the curve $x = a\ cos\theta\ sin\varphi$, $y = a\ sin\theta\ sin\varphi$ and $z = a\ cos\varphi$. (15)

(OR)

(d) Derive the equation satisfying principal curvature and principal direction at a point on a surface.

(15)

5. (a) Derive the Christoffel symbols of first kind. (5)

(OR)

(b) If the lines of curvature are parametric curves then prove that the codazzi equations are $\frac{\partial e}{\partial v} = \frac{1}{2} E_v \left(\frac{e}{E} + \frac{g}{G}\right)$ and $\frac{\partial g}{\partial u} = \frac{1}{2} G_u \left(\frac{e}{E} + \frac{g}{G}\right)$. (5)

(c) Derive the partial differential equation of surface theory. (15)

(OR)

(d) State the Fundamental theorem of Surface Theory and demonstrate it in the case of unit sphere.

(15)

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