LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2022**

UMT 1303 – MATHEMATICS FOR STATISTICS

Date: 01-12-2022   Dept. No. [   ]   Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION – A			
Answer ALL the Questions			
1. **Definition**		**(5 x 1 = 5)**	
a)	Define the differential coefficient of the product of two functions?	K1	CO1
b)	Write the Leibnitz formula for the nth derivative of a product.	K1	CO1
c)	What is the first order partial differential coefficients of $u = \frac{xy}{x+y}$?	K1	CO1
d)	Mention any two types of integration.	K1	CO1
e)	Define definite integral of a function over the limit a to b.	K1	CO1
2. **Choose the correct answer**		**(5 x 1 = 5)**	
a)	The differential coefficient of $\sin(ax + b)$ is _____ (a) $a \sin(ax + b)$ (c) $-a \sin(ax + b)$ (b) $a \cos(ax + b)$ (d) $-a \cos(ax + b)$	K1	CO1
b)	The points of inflexion for the curve $y = x^3 - 9x^2 + 7x - 6$? (a) (3, 39) (b) (-3, 39) (c) (3, -39) (d) (-3, -39)	K1	CO1
c)	$u = \frac{x^3+y^3}{x-y}$ is a homogenous function of x and y of degree _____. (a) 1 (b) 2 (c) 3 (d) 6	K1	CO1
d)	$\int \frac{dx}{a^2+x^2} = $ _____. (a) $\sinh^{-1}\left(\frac{x}{a}\right)$ (c) $\cosh^{-1}\left(\frac{x}{a}\right)$ (b) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (d) $\sin^{-1}\left(\frac{x}{a}\right)$	K1	CO1
e)	Find $\int_1^2 \left(x^2 + \frac{1}{x^2}\right) dx$ is _____. (a) $\frac{14}{5}$ (b) $\frac{17}{6}$ (c) $\frac{11}{6}$ (d) $\frac{4}{5}$	K1	CO1
3. **Fill in the blanks**		**(5 x 1 = 5)**	
a)	The differential coefficient of $cosec\ x$ is _____.	K2	CO1
b)	The range of x for the function $2x^3 - 9x^2 + 12x + 4$ to be a decreasing function is _____.	K2	CO1
c)	If $u = e^{xy}$ then $\frac{\partial^2 u}{\partial x^2}$ is _____.	K2	CO1
d)	Integrating $\frac{dx}{9x^2+4}$ with respect to x is _____.	K2	CO1
e)	The reduction formula for the function $x^n e^{ax}$ is _____.	K2	CO1

4.	**True or False**		**(5 x 1 = 5)**
a)	Differentiation of $e^{\sin^{-1}x}$ with respect to $\sin^{-1}x$ is $e^{\sin^{-1}x}$.	K2	CO1
b)	The function $y = 3x^2 - 2x^3$ is convex downwards for the points $x > \frac{1}{2}$.	K2	CO1
c)	A function $f(x, y)$ attains a maximum value if $rt - s^2 > 0$ and $r < 0$.	K2	CO1
d)	The integration of the function $\sqrt{x^2 + a^2}\, x$ with respect to x is $\frac{1}{2}(x^2 + a^2)^{\frac{3}{2}}$.	K2	CO1
e)	$\int_0^{\pi/2} \sin^6 x \, dx = \frac{5\pi}{32}$.	K2	CO1

SECTION – B

Answer any TWO of the following		**(2 x 10 = 20)**
5.	(a) Discover the differential coefficient of $e^x \sin x \log x$ with respect to x. (b) Solve $\frac{d}{dx}\left(\frac{\sqrt{x}}{2x+3}\right)$.	K3	CO2
6.	If $x(1 + y)^{1/2} + y(1 + x)^{1/2} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$.	K3	CO2
7.	Determine y_n, when $y = \frac{x^2}{(x-1)^2(x+2)}$.	K3	CO2
8.	Apply the appropriate property of definite integral and calculate $\int_0^{\pi/2} \log \sin x \, dx$.	K3	CO2

SECTION – C

Answer any TWO of the following		**(2 x 10 = 20)**
9.	By applying logarithmic differentiation, differentiate $\left(\frac{x^2(x^3-1)}{(2-3x)^5}\right)^{1/3}$ with respect to x.	K4	CO3
10.	Estimate the maxima and minima points of the function $2x^3 - 3x^2 - 36x + 10$.	K4	CO3
11.	State Euler's theorem and analyse whether it is true for the function $u = x^3 + y^3 + z^3 + 3xyz$.	K4	CO3
12.	Integrate $\frac{6x+5}{\sqrt{6+x-2x^2}}$ with respect to x.	K4	CO3

SECTION – D

Answer any ONE of the following		**(1 x 20 = 20)**
13.	If $y = \sin(m \sin^{-1} x)$, then $(1 - x^2)y_2 - xy_1 + m^2 y = 0$ and $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y = 0$, justify.	K5	CO4
14.	Evaluate $\int \frac{x^3}{(x-1)(x-2)} dx$, using partial fractions.	K5	CO4

SECTION – E

Answer any ONE of the following		**(1 x 20 = 20)**
15.	Derive the maximum and minimum values of the curve $u = 2(x^2 - y^2) - x^4 + y^4$.	K6	CO5
16.	Explain the five properties of definite integration and hence show that $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) \, d\theta = \frac{\pi}{8}\log 2$.	K6	CO5

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