LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

UMT 1501 – ALGEBRA

(19, 20 BATCH)

Date: 24-11-2022   Dept. No.   Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

PART - A

(Answer all Questions)   **(10×2=20)**

1. Form a quadratic equation, given that $-2 + \sqrt{-7}$ is a root.

2. Find the sum of the roots of equation $x^4 - 2x^3 + 4x^2 + 6x - 2 = 0$.

3. Find the number of real roots of the equation $x^3 + 18x - 6 = 0$.

4. Show that the equation $x^5 - 6x^2 - 4x + 5 = 0$ cannot have more than one negative root.

5. Find the value of $\frac{e+e^{-1}}{2}$ and $\frac{e-e^{-1}}{2}$

6. Write the expansion of $log(1 + x)$ and $-log(1 - x)$.

7. State Cayley-Hamilton theorem.

8. Find the characteristic equation of the matrix $\begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$.

9. Find the number of integers less than and prime to 729.

10. Find the number and the sum of all divisors of 360.

PART - B

(Answer any 5 Questions)   **(05×8=40)**

11. Find $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$, where α, β, γ are the roots of the equation
$x^3 + 2x^2 - 3x - 1 = 0$.

12. Diminish the roots of the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2 and write the transformed equation.

13. Find the sum to infinity of the series $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \ldots\ldots$

14. Find the sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \frac{1+2+2^2+2^3}{4!} + \ldots..$

15. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 2 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find its inverse.

16. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -1 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

17. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

18. State and prove Fermat's theorem.

PART - C

(Answer any 2 Questions)

19. a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$, whose roots are in hormonic progression. **(10 Marks)**

   b) Solve the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$. **(10 Marks)**

20. a) Calculate the roots of the equation $x^3 - 3x + 1 = 0$ to two places of decimal which lies between 1 and 2 by using Horner's method. **(10 Marks)**

   b) Solve the equation $x^3 - 9x^2 + 108 = 0$ using Cardon's method. **(10 Marks)**

21.a) Show that $log\sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \cdots.$ **(10 Marks)**

   b) State Wilson's theorem and prove that $18! + 1$ is divisible by 437. **(10 Marks)**

22. Diagonalize the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$

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