

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2022

UMT 3502 – DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Date: 03-12-2022

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL the Questions

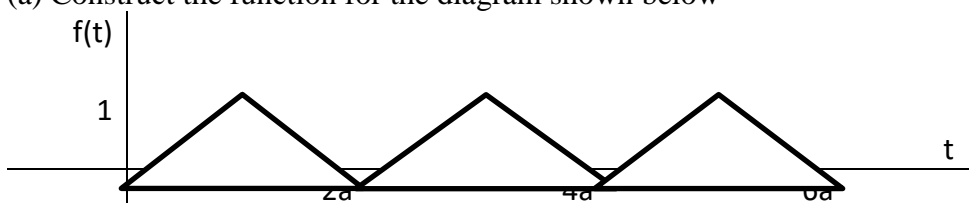
1.	Answer the following:		(5 x 1 = 5)
a)	Write the solution of the equation $\frac{dy}{dx} = \frac{y+a}{x-b}$.	K1	CO1
b)	Identify the type of the equation $(1 + 2x)^2 \frac{d^2y}{dx^2} + (1 + 2x) \frac{dy}{dx} + y = 8(1 + 2x)^2$.	K1	CO1
c)	Define complete integral.	K1	CO1
d)	Give an expression for $L(f''(t))$.	K1	CO1
e)	What is $L^{-1}\left(\frac{1}{s} - \frac{1}{s+10}\right)$?	K1	CO1
2.	Fill in the blanks		(5 x 1 = 5)
a)	The order of the ODE $(x + 1) \frac{dy}{dx} + 1 = 2e^{-y}$ is _____.	K1	CO1
b)	Second order linear ODE with variable coefficients is also known as _____.	K1	CO1
c)	The solution for the equation $pq = 1$ is _____.	K1	CO1
d)	In Laplace transform s is called as _____.	K1	CO1
e)	$L^{-1}\left[\frac{s}{(s-1)^3}\right]$ is _____.	K1	CO1
3.	Choose the correct answer for the following		(5 x 1 = 5)
a)	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} + 2\frac{dy}{dx} + y = 0$ is (i) 1 (ii) 2 (iii) 3 (iv) 4	K2	CO1
b)	The particular integral of the differential equation is $(D^2 - 4)y = e^{3x}$ is (i) $\frac{e^{3x}}{4}$ (ii) $\frac{e^{3x}}{5}$ (iii) $\frac{e^{4x}}{4}$ (iv) $\frac{e^{3x}}{3}$	K2	CO1
c)	The solution of $z = px + qy + pq$ is (i) $z = p + q + pq$ (ii) $z = ax + by + pb$ (iii) $z = pa + qb + ab$ (iv) $z = cx + dy + cd$	K2	CO1
d)	$L\left(\frac{t^5}{3}\right) =$ (i) $\frac{10}{s^6}$ (ii) $\frac{40}{s^5}$ (iii) $\frac{30}{s^6}$ (iv) $\frac{40}{s^6}$		

e)	$L^{-1}\left(\frac{a}{s^2+a^2}\right)$ is (i) $\sin at$ (ii) $\cos at$ (iii) $\sinh at$ (iv) $\cosh at$	K2	CO1
4.	Say TRUE or FALSE 1 = 5)	(5 x	
a)	Linear ODE is a particular case of Bernoulli's equation.	K2	CO1
b)	The complementary function and general solution are different for $(D^2 - 2mD + m^2)y = 0$.	K2	CO1
c)	Particular integral is a singular integral.	K2	CO1
d)	It is a necessary condition that a function should be of exponential order to have Laplace transform.	K2	CO1
e)	Laplace technique is used to evaluate certain integrals.	K2	CO1
SECTION B			
Answer any TWO of the following in 100 words 20)			(2 x 10 =
5.	A body of mass m falling from rest is subject to force of gravity and air resistance proportional to the square of velocity. If it falls through a distance x and possess a velocity v at that instant, prove that $\frac{2kx}{m} = \log \frac{a^2}{a^2-v^2}$ where $mg = ka^2$. (10 marks)	K3	CO2
6.	Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$. (10 marks)	K3	CO2
7.	Solve the equation $p + q = x + y$. (10 marks)	K3	CO2
8.	Find $L\left(\frac{\sin^2 t}{t}\right)$. (10 marks)	K3	CO2
SECTION C			
Answer any TWO of the following in 100 words 20)			(2 x 10 =
9.	Solve the non-homogeneous equation $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$. (10 marks)	K4	CO3
10.	Reduce the equation $(5 + 2x)^2 \frac{d^2y}{dx^2} - 6(5 + 2x) \frac{dy}{dx} + 8y = 6x$ to a linear homogeneous equation and hence solve. (10 marks)	K4	CO3
11.	Find the Laplace transform of rectangular wave given by $f(t) = \begin{cases} 1, & \text{if } 0 < t < b \\ -1, & \text{if } b < t < 2b \end{cases}$. (10 marks)	K4	CO3
12.	Determine $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$. (10 marks)	K4	CO3
SECTION D			
Answer any ONE of the following in 250 words 20)			(1 x 20 =
13.	(a) Solve $(D^2 + 3D + 2)y = e^{-x} + x^2 + \cos x$. (10 marks)	K5	CO4
	(b) Find the general solution of $(y + z)p + (z + x)q = x + y$. (10 marks)		

14.	(a) Determine Laplace transform of $(\sin at - at \cos at)$. (10 marks)	K5	CO4
	(b) Evaluate $\int_0^{\infty} \frac{e^{-t} - e^{-2t}}{t} dt$. (10 marks)		

SECTION E

Answer any ONE of the following in 250 words (1 x 20 = 20)

15.	(a) Construct the auxiliary equation for Charpit's method and hence determine the solution for $p^2 + q^2 - 2px - 2qy + 1 = 0$. (12 marks)	K6	CO5
	(b) Reduce the equation $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$ to the linear form and solve. (8 marks)		
16.	(a) Construct the function for the diagram shown below 	K6	CO5
	(b) Find the function y if given $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$. (2+18 marks)		

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