LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **NOVEMBER 2022**

UMT 4501 – REAL ANALYSIS-I

Date: 26-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the questions: **(10 × 2 = 20 Marks)**

1. Define a bijection on ***R***. Give an example.
2. State Cantor's theorem.
3. Write down the triangle inequality in ***R***.
4. Define a bounded sequence in ***R***.
5. State the Archimedean property.
6. Define a nested interval in ***R***.
7. Find the limit of the sequence $\left(\frac{1}{n^2}\right)$.
8. Write the Cauchy convergence criterion.
9. State the comparison test.
10. Define an absolutely convergent series.

PART – B

Answer any FIVE of the following: **(5 × 8 = 40 Marks)**

11. Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.
12. Let a, b, c be any elements of ***R.*** Then prove the following.
    a) If $a > b$ and $b > c$, then $a > c$
    b) If $a > b$, then $a + c > b + c$
13. Prove that there exists a positive real number x such that $x^2 = 2$.
14. State and prove the Squeeze theorem.
15. Define rearrangement. State and prove the rearrangement theorem.
16. Justify the statement, "If a series in ***R*** is absolutely convergent, then it is convergent".
17. Explain the principle of mathematical induction.
18. State and prove Bolzano Weierstrass theorem.

PART – C

Answer any TWO of the following: **(2 × 20 = 40 Marks)**

19. a) State and prove the De Morgan laws.

    b) Determine the set $A = \{x \in \boldsymbol{R} : x^2 > 3x + 4\}$. (10 + 10)

20. a) Prove that the set $\boldsymbol{R}$ of all real numbers is uncountable.

    b) Prove that a convergent sequence of real numbers is bounded. (10 + 10)

21. a) State ratio, Raabe's, Dirichlet and Abel's tests.

    b) Test the convergency of the sequences (n) and $((-1)^n)$. Justify. (10 + 10)

22. a) State and prove the alternating series test.

    b) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

    (10 + 10)

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