LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

UMT 5503 – DISCRETE MATHEMATICS

Date: 28-11-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

SECTION – A [10 X 2 = 20 MARKS]

1) Write each of the following is symbolic form where P: Satish is poor and Q: Satish is happy.
   (a) Satish is poor but not happy.
   (b) Satish is either happy or poor.
2) Write the duals of (a) $(P \vee Q) \wedge R$ (b) $(P \wedge Q) \vee T$
3) Symbolize the expression
   "All the world loves a lover"
4) Is the following argument valid? If this number is divisible by 4, then it is divisible by 2. This number not divisible by 2. Therefore this is not divisible by 4.
5) Define Monoid with an example.
6) Define Semigroup homomorphism
7) Define Lattice.
8) Let D_{24} be the set of divisors of 24 and the relation ≤ is $a \le b \; if \; a/b$. Draw the Hasse diagram for D_{24}.
9) Define Sub-Boolean algebra.
10) Define direct product of Boolean algebra.

SECTION – B [5X8=40 MARKS]

11) Construct the truth table for $(p \wedge q) \vee (\urcorner p \wedge q) \vee (p \wedge \urcorner q)$
12) Show that the formula $Q \vee (P \wedge \urcorner Q) \vee (\urcorner P \wedge \urcorner Q)$is a tautology.
13) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\urcorner R \vee P \; and \; Q$.
14) Find any commutative monoid $(M,*)$, prove that the set of idempotent elements of M forms a submonoid.
15) If $(L, \le)$ is a lattice in which meet and join are denoted by $* \; and \; \oplus$. Then prove that for any $a, b \in L$
   (i) $a \le b$ (ii) $a * b = a$ (iii) $a \oplus b = b$ are equivalent.
16) Let $(L, \le)$ be a lattice. Then for any $a, b, c \in L$ Prove that following inequality holds
   $$a \le c <=> a \oplus (b * c) \le (a \oplus b) * c)$$
17) State and prove Demorgan's law of Boolean algebra.
18) Obtain the values of the Boolean forms $x_1 * (x_1' * x_2)$, $x_1 * x_2$, $x_1 \oplus (x_1 * x_2)$ over the ordered pair of the two element boolean algebra.

SECTION – C [2 x 20 = 40 MARKS]

19) a) Obtain the principal conjunctive normal form and principal disjunctive normal form of the formula S given by $(\urcorner P \rightarrow R) \wedge (Q \rightleftarrows P)$

b) If $H_1, H_2, \ldots\ldots\ldots H_m$ and P imply Q then $H_1, H_2, \ldots\ldots\ldots H_m$ imply $P \rightarrow Q$.

20) a) Show that the following premises are inconsistent

(1)If Jack misses many classes through illness, then he fails high school.

(2) If Jack fails high school, then he is uneducated.

(3) If Jack reads a lot of books, then he is not uneducated.

(4) Jack misses many classes through illness and reads a lot of books.

b) Let $(S,*), (T,\Delta) and\ (V,\oplus)$ be semigroups and $g : S \rightarrow T\ and\ h: T \rightarrow V$ be semigroup homomorphisms. Prove that $(h_o g) : S \rightarrow V$ is a semigroup homomorphism from $(S,*) to\ (V,\oplus)$

21) State and prove the properties of Lattices.

22) a)Write the Boolean expression in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3 :

a) $x_1 * x_2$ (b) $x_1 \oplus x_2$ (c) $(x_1 \oplus x_2)' * x_3$

b) Define Boolean expression.

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