LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2022**

UMT 6501 – COMPLEX ANALYSIS

Date: 01-12-2022 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL Questions: **(10 x 2 = 20)**

1. Show that the function $f(z) = Re\, z$ is nowhere differentiable.
2. Find the singular points of the function $f(z) = \frac{z^3+4}{(z^2-3)\,(z^2+1)}$.
3. Define harmonic function.
4. Evaluate $\int_C \frac{dz}{z-1}$ where C is the circle $|z-2| = 3$.
5. State Cauchy – Goursat's theorem for a continuous function *f*.
6. Give the Maclaurin's series of e^z.
7. State maximum modulus principle.
8. What is an essential singularity?
9. Find the residue of $\frac{z^2}{z^2+a^2}$ at $z = ai$.
10. Define linear fractional transformation.

PART – B

Answer any FIVE Questions: **(5 x 8 = 40)**

11. Show the function $f(z) = \sqrt{|xy|}$ is not differentiable but satisfies the Cauchy – Riemann equations.
12. State and prove the polar form of the Cauchy – Riemann equations.
13. Find the harmonic conjugate of $u(x,y) = y^3 - 3x^2y$.
14. Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Prove that $\left|\int_C \frac{z+4}{z^3-1}\,dz\right| \le \frac{6\pi}{7}$.
15. State Liouville's theorem and deduce the Fundamental theorem of algebra.
16. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ in a Laurent's series in (i) $1 < |z| < 2$ and (ii) $|z| > 2$.
17. State and prove Cauchy residue theorem.
18. Find the bilinear transformation which maps $z_1 = -1, z_2 = 0$ and $z_3 = 1$ onto the points $w_1 = -i, w_2 = 1$ and $w_3 = i$.

PART – C

Answer any TWO Questions: **(2 x 20 = 40)**

19. State and prove the necessary and sufficient condition for a function $f(z)$ to be differentiable at a point.

20. (a) If $w(t)$ is a piecewise continuous complex valued function defined on an interval $a \leq t \leq b$, then prove that $\left|\int_a^b w(t)dt\right| \leq \int_a^b |w(t)|dt$.

    (b) Evaluate $\int_C \frac{\exp(2z)}{z^4}\, dz$, where C is the positively oriented unit circle $|z| = 1$.

21. (a) State and prove Taylor's Theorem.

    (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\, sin\theta}$ using method of contour integration.

22. (a) State and prove Rouche's theorem.

    (b) Determine the value of the integral $\int_C \frac{5z-2}{z(z-1)} dz$ using residue theorem where C is the circle $|z| = 2$ described counter clockwise.

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