LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – **APRIL 2016**

PH 4504/PH 4502/PH 6604 – MATHEMATICAL PHYSICS

Date: 20-04-2016   Dept. No. [   ]   Max. : 100 Marks
Time: 09:00-12:00

PART – A

Answer ALL questions   **(10 x 2 = 20 Marks)**

1. Find the real and imaginary part of $\frac{1-i}{2+i}$.
2. Check whether $f(z) = Re\,z + Im\,z$ is analytic or not.
3. Show that $\int_c f(z)dz$ is independent of path followed if $f(z)$ in analytic.
4. Evaluate $\int_{-i}^{-i} \frac{dz}{z}$.
5. Find the value of c if $u(x,t) = e^{-2kt}\cos 8x$ is a solution of $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial^2 x}$.
6. Write down the two dimensional wave equations.
7. Define inverse Fourier transform of a function.
8. State convolution theorem.
9. Give the formula for forward difference operator and shift operator.
10. Using Trapezoidal rule, Evaluate $\int_0^{-1} f(x)dx$

x	0	0.25	0.50	0.75	1.00
f(x)	1.000	0.800	0.667	0.571	0.500

PART – B

Answer any FOUR questions   **(4 x 7.5 =30 Marks)**

11. Show that
    (a) $\cos z = \cos x \cosh y - i\,\sin x\,\sinh y$
    (b) $\sinh z = \sinh x\,\cos y + i\,\cosh x\,\sin y$

12. Using Cauchy's integral formula, evaluate $\int_c \frac{\sin z}{\left(z-\frac{i\pi}{2}\right)^4}\,dz$ where C is the circle $|z| = 2$.

13. Derive the wave equation for a vibrating string.

14. Find the Fourier cosine transform of e^{-kx} where $k > 0$.

15. Find the value of y at $x=0.23$ from the following table using Newton's forward interpolation formula

at x	0.20	0.22	0.24	0.26	0.28	0.30
y	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

16. Find the value of $y(0.4)$ for $y' = 1 + y^2, y(0) = 1, h = 0.1$ using Euler's method.

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PART – C

Answer any FOUR questions: $(4 \times 12.5 = 50$ **Marks)**

17. **Verify that** $u = x^3 - 3xy^2$ **is harmonic. Find a harmonic conjugate function** v **and hence find the analytic function** $f(z)$. $(5+5+2.5)$

18. **(a) State and prove Cauchy's integral theorem.**

    (b) Evaluate $\oint_C \frac{z^2 + 4\, dz}{z^2 - 4}$ **in counter clockwise where C: |z-1|=2.** $(7.5+5)$

19. **Derive the D'Alembert's solution for the wave equation.**

20. **(a) If** $F(s)$ **is the Fourier transform of** $f(t)$, **show that** $F\{t\, f(t)\} = -iF'(s)$.

    (b) A semi-infinite solid $x > 0$ **is initially at temperature zero. At time t=0, a constant temperature** u_o **is applied and maintained at the face** $x = 0$. **Find the temperature at any point of the solid and at any time** $t > 0$. $(6+6.5)$

21. **Deduce Lagrange interpolation formula for unequal intervals and using it find** $y(10)$ **from the following table:**

x	5	6	9	11
y	12	13	14	16

$(6.5+6)$

22. **(a) Derive Cauchy-Riemann equations for a function** $f(z)$ **to be analytic.** (7.5)

    (b) Find the real and imaginary parts of $exp\,(z^2)$. (5.0)

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