# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.**DEGREE EXAMINATION – **PHYSICS**

SECONDSEMESTER – APRIL 2018

**PH 2812- MATHEMATICAL PHYSICS**

Date: 25-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

**Part - A**

Answer **ALL** questions (10 x 2 =20)

1. Check whether $f(z) = Re\ (z^2)$ is analytic or not.
2. State Taylor's theorem.
3. Define Dirac Delta function. What is its Laplace transformation?
4. State convolution theorem.
5. Find L$\{e^{at}\ sin\ bt\}$.
6. Define inverse Fourier cosine and sine transformation.
7. Write the orthonormal property of Legendre polynomials.
8. State the condition for which the differential equation is Sturm-Liouville type.
9. Distinguish between Abelian group from cyclic group.
10. What is homomorphism?

**Part –B**

Answer any **FOUR** questions (4 x 7.5 = 30)

11. State and prove Cauchy's theorem.
12. Solve the initial value problem $\frac{d^2y}{dt^2} + 25y = 10\cos t\, , y(0) = 2, y'(0) = 0$ by the Laplace transform.
13. Solve two-dimensional wave equation.
14. Obtain Fourier expansion for the function, $\mathrm{f(x)} = \frac{1}{2}(\pi - \mathrm{x}), 0 < x < 2\pi$.
15. Derive the recurrence relations (i)$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

    (ii)$\frac{d}{dx}[x^{n-1} J_n(x)] = -x^{-n} J_{n+1}(x)$
16. Develop transformation matrix for rotation operation. Predict the number of rotational and vibrational modes of linear molecule $CO_2$.

**Part –C**

Answer any **FOUR** questions (4 x 12.5 = 50)

17. (i) Find the Taylor's series to represent $\frac{z^2-1}{(z+2)(z+3)}$ when $|z| < 2$.

    (ii) Derive Cauchy-Riemann equation for a function to be analytic.

18. i) Evaluate $\int_c \frac{f(z)}{z-2}\, dz$ c: $|z| = 5$.

(ii) $f(z) = u(x, y) + iv(x, y)$ is an analytic function and $u(x, y)= \frac{\sin 2x}{\cos h\, 2y + \cos 2x}$ find $f(z)$.

19. Obtain the general solution of partial differential equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ with the boundary condition $y(0, t) = 0;\ y_x(L, t) = 0;\ y(x, 0) = f(x), y_t(x, 0) = 0, |y(x, t)| < M$.

20. Derive the orthogonality relation for Laguerre's polynomials.

21. (a) Obtain the transformation matrices of the symmetry elements i) for the axis of symmetry and ii) for the improper axis of symmetry

(b) Enumerate and explain the symmetry elements of $H_2O$ and $NH_3$ molecules.

22. Solve Legendre's differential equation by Frobenius power series method.

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