LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

SECOND SEMESTER – APRIL 2022

UPH 2502 – MATHEMATICAL PHYSICS - I

Date: 18-06-2022   Dept. No.   Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

PART – A

Q. No.   Answer **ALL** Questions   **(10 x 2 = 20 Marks)**

1. Express in polar form: $1 - \sqrt{2} + i$
2. Simplify the following: (a) i^{49} and (b) i^{178}.
3. Test if the function $\sin z$ is analytic.
4. Prove that $(\cos hx - \sin hx)^n = \cos h\, nx - \sin h\, nx$
5. Find the area of the parallelogram whose adjacent side are $2i - 4j + k$ and $i - 6j - 4k$.
6. Determine λ such that $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - 4\hat{k}$ and $\vec{c} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$ are coplanar.
7. Evaluate grad , if $\varphi = \log(x^2 + y^2 + z^2)$.
8. Find the torque of a force $7\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ about the origin which acts on a particle whose position vector is $\hat{\imath} + \hat{\jmath} - \hat{k}$.
9. Write Dirichlet's conditions.
10. What are odd and even functions?

PART – B

Answer any **FOUR** Questions   **(4 x 7.5 = 30 Marks)**

11. Find the square root of the complex number $5 + 12i$.
12. Determine whether $\frac{1}{z}$ is analytic or not?
13. Derive Cauchy's Integral formula.
14. Find the values of a, b, c so that the function $\vec{f} = (x + 2y + az)\hat{\imath} + (bx - 3y - 3z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.
15. Find the directional derivative of $\frac{1}{r}$ in the direction $\vec{r}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.
16. Find the Fourier series for the periodic function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ $f(x + 2\pi) = f(x)$.

PART – C

Answer any **FOUR** Questions   **(4 x 12.5 = 50 Marks)**

17. Evaluate the following integral using Cauchy's integral formula $\int_c \frac{z}{(z^2 - 3z + 2)}\, dz$ Where 'c' is the circle $|z - 2| = \frac{1}{2}$.
18. Find the complex number z is if $\arg(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$.
19. State and Prove Cauchy's Integral Theorem.
20. If $\vec{u} = yz\,\hat{\imath} + zx\,\hat{\jmath} + xy\,\hat{k}$ and $f = xyz$ find $Curl(f\vec{U})$.
21. Find the divergence and curl of $\vec{v} = (xyz)\,\hat{\imath} + (3x^2y)\,\hat{\jmath} + (xz^2 - y^2z)\,\hat{k}$ at (2, -1, 1).
22. Find the Fourier series to represent $f(x) = x\sin x$ for $0 < x < 2\pi$.

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