LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRST SEMESTER – **NOVEMBER 2016**

16PPH1MC04/PH 1820 – MATHEMATICAL PHYSICS - I

Date: 09-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART A

Answer ALL questions **(10 x 2 = 20)**

1. Write the algorithm of modified Euler's method to solve differential equations.
2. Sketch the graph $y = \sin x$.
3. Express $\frac{1+i}{1-i}$ in the form of $f(z) = a + ib$.
4. Give each an example of single valued and multi valued functions.
5. Show that vectors $V_1 = (1,-2,1)$; $V_2 = (2,1,-1)$ and $V_3 = (7,-4,1)$ are linearly dependent in R^3
6. The 2 x 2 matrices $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$ and $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$ span the vector space M_{22} of all 2 x 2 matrices with real entries.
7. Show that $\beta(m,n) = \beta(n,m)$ where β stands for Beta function.
8. Evaluate $\int_0^{\pi/2} e^{-x^2} dx$ using Gamma function.
9. Evaluate a) $\delta_j^i . \delta_k^j . \delta_l^k$ b) $\delta_j^i A^j$.
10. Prove that the contracted tensor A_j^i is a scalar.

PART B

Answer any FOUR questions **(4 x 7.5 = 30)**

11. Solve the system of equations by Gauss-Seidel method.
    $6x + y + z = 105$; $4x + 8y + 3z = 155$; $5x + 4y - 10z = 65$.
12. In the given region of $|z| = 13$, evaluate $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz$.
13. Show that a set $s = \{(1,2,1), (2,1,0), (1,-1,2)\}$ forms a basis in R^3.
14. Express the conjugate metric tensor in spherical coordinates.
15. Show that Bessel's generating function is represented by $e^{\frac{x}{2}(t-t^{-1})} = \Sigma_{n=-\infty}^{\infty} J_n(x) t^n$.
16. Find the residue of $\frac{z^3}{(z-1)(z-2)(z-3)}$ where the region is defined by the circle $|z|=2$.

PART C

Answer any FOUR questions **(4 x 12.5 = 50)**

17. Using Gauss-Seidel method, starting with initial approximation $x_1 = 0.3, x_2 = -0.8$ and $x_2 = 0.3$ evaluate the system of equations $2x_1 - x_2 + 2x_3 = 3$; $x_1 + 3x_2 + 3x_3 = -1$; $x_1 + 2x_2 + 5x_3 = 1$.

18. Evaluate $\int_{-\infty}^{+\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$ using contour integration.

19. Using Gram-Schmidt orthogonalization process construct an orthonormal basis for R^3 with standard inner product out of the basis $\{v_1, v_2, v_3\}$, where $v_1 = (1,0,1)$, $v_2 = (1,0,-1)$, $v_3 = (0,3,4)$.

20. Using Frobenius power series method, obtain the solution of Legendre's differential equation.

21. a) Find the metric tensor and the expression for the line element in cylindrical coordinates.

    b) Show that a symmetric tensor of the second order has only $\frac{n(n+1)}{2}$ different components.

    c) Show that if a tensor is symmetric with respect to two indices in any coordinate system, it will remain symmetric with respect to these two indices in any other coordinate system.

22. Compute the real root of $xlogx - 1.2 = 0$ using Newton - Raphson method.

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