

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**B.Sc. DEGREE EXAMINATION – PHYSICS****THIRD SEMESTER – NOVEMBER 2016****PH 3506 – MATHEMATICAL PHYSICS**

Date: 04-11-2016

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART – A**(10 x 2 = 20 Marks)****Answer ALL the questions:**

1. Find the principal value of i^i where $i = \sqrt{-1}$.
2. State Cauchy's integral theorem.
3. Integrate $xy \, dx \, dy$ between $x = 0$ to $x = 1$, $y = 0$ to $y = 1$.
4. Write the heat equation.
5. Determine the fundamental period of $\cos \pi x$ and $\sin x$.
6. What is Fourier's integral?
7. Given $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, find $A^T A$, where A^T is the transpose of matrix A.
8. If the det value of a 2×2 matrix B is 4. What is the det value of 2B?
9. Write Newton's forward interpolation formula for equally spaced data.
10. Given $y' = 1 + y$ with $y(0) = 1$ and step size as 0.2, find the value of $y(0.2)$ using Euler's method.

PART – B**(4 x 7.5 = 30 Marks)****Answer any FOUR questions:**

11. Determine the harmonic conjugate function of $u = \sin x \cosh y$ and its corresponding analytic function.
12. State and prove Green's theorem in a plane.
13. Find the Fourier series of the function $f(t) = \begin{cases} 0, & \text{if } -\frac{\pi}{\omega} < t < 0 \\ E \sin \omega t, & \text{if } 0 < t < \frac{\pi}{\omega} \end{cases}$
14. Show that $N = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ satisfies Cayley-Hamilton theorem.
15. Find the Lagrange interpolating cubic polynomial for the following

x	0	3	5	6
y(x)	-6	6	104	196

and compute $y(4)$.

16. Using Trapezoidal rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$ by dividing the range into 10 equal parts.

Answer any FOUR questions:

17. (i) Find the real and imaginary parts of $\tanh (x + iy)$.

(ii) Integrate $\oint \frac{(1+2z) \cos z}{(2z-1)^2} dz$ counterclockwise around the circle $|z| = 1$. (7.5+5)

18. Given $\vec{v} = y^2 \hat{i} + z^2 \hat{j} + x^2 \hat{k}$ and $\phi = x + y + z$, find the following

(i) $\text{curl}(\text{grad } \phi)$ (ii) $\text{div}(\text{curl } \vec{v})$ (iii) $\text{curl}(\phi \vec{v})$ (4+4+4.5)

19. Obtain the Fourier Cosine series and Fourier Sine series of the function

$$f(x) = \begin{cases} \frac{2}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$

20. Diagonalize the matrix $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

21. Find the solution to four decimals of the following system of simultaneous equations by Gauss-Seidel iterative method.

$$\begin{aligned} 10x + 2y + z &= 9 \\ 2x + 20y - 2z &= -44 \\ -2x + 3y + 10z &= 22 \end{aligned}$$

22. (i) State and prove Cauchy's integral formula.

(ii) Find the directional derivative of $f = xyz$ at the point $(-1, 1, 3)$ in the direction of

$$\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}. \quad (7.5+5)$$
