LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2023**

PPH3MC01 – QUANTUM MECHANICS I

Date: 30-10-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1.	**True or False**
a)	The set of functions $\{1,\ x, 3x,\ x^3\}$ are linearly independent
b)	If $\hat{n}$ is the number operator, $[\hat{a}, \hat{n}] = -\hat{a}$
c)	The spin operator $\hat{\sigma}_x$ acts as flip operator on the eigen-kets of $\hat{\sigma}_z$
d)	If the first order perturbation correction to an energy level is zero, the first order correction to eigen-vectors must also be zero.
e)	The difference in scattering amplitude between a point scatterer and an extended one can be expressed using form factor.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2.	**Definitions**
a)	Linear independence of N vectors, $\|i\rangle, i \to 1,2,3 \ldots. N$.
b)	Fock states
c)	Selection rules in the addition of two angular momenta, $\vec{J}_1$ and $\vec{J}_2$.
d)	Zeeman effect
e)	Screened Coulomb potential
	SECTION B – K3 (CO2)
	Answer any THREE of the following in 300 words **(3 x 10 = 30)**
3.	i. If $\hat{O}$ is defined as $\hat{O}\psi = \frac{d\psi}{dx} + \psi$, check if $\hat{O}$ is a linear operator. (2) ii. Obtain the matrix form of a linear operator $\hat{A}$ in the complete set of vectors $\{\|i\rangle; i = 1, \ldots. N\}$. (2) iii. Obtain the matrix form of the operator $\hat{R}\left(\hat{k}\,\frac{\pi}{4}\right)$ which is the anticlockwise rotation of the three dimensional Cartesian coordinate system through 90^0 (6)
4.	Setup the Schrodinger wave equation for a stream of particles of mass m and constant energy E, incident on a step potential of height V_0 at $x = 0$. Calculate the general expressions for the transmission and reflection coefficients and discuss the limiting cases of energy, in comparison to the height of the step.
5.	i. Consider a system with two angular momenta $\overrightarrow{J_1}$and $\overrightarrow{J_2}$ where $j_1 = j_2 = 1$. Find the two complete set of kets that can span the Hilbert space of the composite system. (Hint: Apply the selection rules for the addition of angular momenta.) (5) ii. If $\vec{A}$ and $\vec{B}$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are arbitrary vectors, show that, $(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{A}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A} \times \vec{B})$ (5)

6.	Derive the general expressions for the first order and second order corrections to the non-degenerate energy levels and the wave functions of a perturbed system.
7.	Derive the Breit-Wigner formula for scattering by an attractive square potential well making use of the phase shift analysis.
	SECTION C – K4 (CO3)
	Answer any TWO of the following in 500 words (2 x 12.5 = 25)
8.	The Hamiltonian of a two level system is given by $\widehat{H} = a(\lvert 1\rangle\langle 1\rvert - \lvert 2\rangle\langle 2\rvert + \lvert 1\rangle\langle 2\rvert + \lvert 2\rangle\langle 1\rvert)$. Determine the energy eigenvalues and the normalized eigenkets of this Hamiltonian
9.	A coherent beam of particles of mass m, energy E and unit intensity falls on a potential barrier of height V_0 and width a. Derive an expression for the transmission coefficient T. Apply this result to explain the α −decay problem.
10.	Introduce the angular momentum ladder operators and demonstrate how they affect $\lvert l, m\rangle$. Hence deduce the matrix form of the angular momentum operators for $l = 1$.
11.	Applying perturbation theory for non-degenerate levels, find the first order energy correction to the n^{th} level of an oscillator with its potential energy given by $V(x) = \frac{1}{2}\, m\omega^2 x^2 + ax^3 + bx^4$.
	SECTION D – K5 (CO4)
	Answer any ONE of the following in 750 words (1 x 15 = 15)
12.	With a neat sketch describe Stern and Gerlach experiment. Develop the theory of spin angular momentum and obtain the Pauli spin operators.
13.	Formulate the perturbation theory of degenerate levels and apply it to the spin-orbit angular momentum coupling giving rise to Zeeman effect.
	SECTION E – K6 (CO5)
	Answer any ONE of the following in 1000 words (1 x 20 = 20)
14.	Reduce a two-body problem into a single body problem. Using it, setup the Schrodinger wave equation for the Hydrogen atom. Applying the variable separable method, convert the partial differential equation in three variables into three ordinary differential equations and solve them to get the normalized eigen-functions and the corresponding eigenvalues. What are quantum numbers that emerge during this process and what is their significance?
15.	Consider two angular momenta $\vec{J}_1$ and $\vec{J}_2$ with eigenkets $\lvert j_i, m_i\rangle$ satisfying equations $\widehat{J_i^2}\lvert j_i, m_i\rangle = j_i(j_i + 1)\hbar^2\lvert j_i, m_i\rangle$, $\hat{J}_z\lvert j_i, m_i\rangle = m_i\hbar\lvert j_i, m_i\rangle$; $i \rightarrow 1,2$. Construct the two complete set of eigen-vectors of the composite system of the two angular momenta. Obtain the transformation coefficients (Clebsch-Gordan coefficients) connecting the two bases. Derive these coefficients for the case $j_1 = 1,\ j_2 = \frac{1}{2}$

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