## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## M.Sc. DEGREE EXAMINATION - CHEMISTRY <br> FIRST SEMESTER - NOVEMBER 2022

## PCH1MC03 - QUANTUM CHEMISTRY AND GROUP THEORY

Date: 28-11-2022
Time: 01:00 PM - 04:00 PM
Dept. No. $\square$

Max. : 100 Marks

| SECTION A |  |  |  |
| :---: | :---: | :---: | :---: |
| Answer ALL the questions |  |  |  |
| 1 | Answer the following | ( $5 \times 1=5$ ) |  |
| a) | Mention the limits and Laplacian for spherical coordinates. | K1 | CO1 |
| b) | How many degenerate energy levels lie in $11 \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}$ for a particle in a cubic box of length L? | K1 | CO1 |
| c) | Write the Hamiltonian for $\mathrm{H}_{2}{ }^{+}$ion. | K1 | CO1 |
| d) | Identify the number of reflection planes present in a molecule of $\mathrm{C}_{4 \mathrm{v}}$ point group. | K1 | CO1 |
| e) | Mention the operator involved in resonance integral. | K1 | CO1 |
| 2 | Answer the following | ( $5 \times 1=5$ ) |  |
| a) | Find the accelerating potential for an electron with de Broglie wavelength of 5 A. | K2 | CO1 |
| b) | Predict the value of $\mathrm{H}_{0}(\mathrm{q})$. | K2 | CO1 |
| c) | Write the Slater determinant for the ground state configuration $1 s^{2}$. | K2 | CO1 |
| d) | What is the point group of $\mathrm{BFCl}_{2}$ molecule? | K2 | CO1 |
| e) | Mention the significance of coulomb integral. | K2 | CO1 |
| SECTION B |  |  |  |
|  | Answer any THREE of the following 30) | $(3 \times 10=$ |  |
| 3 | (a) State and explain the postulates of quantum mechanics. <br> (b) Which of the following operators are linear? ( ) ${ }^{2}$ and $\mathrm{d}^{2} / \mathrm{dx}^{2}$. Justify. | K3 | CO 2 |
| 4 | Write the Schrödinger wave equation for rigid rotator in terms of spherical angular coordinates. Using the method of separation, separate them into two independent variables such as $\mathrm{P}(\theta)$ and $\mathrm{Z}(\Phi)$ and obtain the solution for $\Phi$ equation. | K3 | CO2 |
| 5 | (a) For a particle in an infinitely deep one-dimensional potential box of length L, apply the trial wave function $\psi=\mathrm{N} x\left(\mathrm{~L}^{2}-x^{2}\right)$ to calculate the energy and obtain the percentage of error. <br> (b) State the Pauli's exclusion principle for ground state electronic configuration of lithium atom. | K3 | CO 2 |
| 6 | a) List the symmetry elements and operations of cyclopropane molecule. <br> b) Obtain the matrix representation for the refelection operation $\sigma_{y z}$. | K3 | CO2 |
| 7 | a) Obtain the reducible representation relating to the prediction of hybridisation scheme in $\mathrm{CH}_{3} \mathrm{Cl}$ molecule. <br> b) Evaluate the overlap integral $\mathrm{S}_{12}$ in the formation of $\mathrm{H}_{2}{ }^{+}$ion when the distance of separation between the nuclei of two 1 S orbitals is $1.32 \AA$. Given the first Bohr radius 0.529 Å. | K3 | CO2 |
| SECTION C |  |  |  |
|  | Answer any TWO of the following (2 | x $12.5=25$ ) |  |
| 8 | (a) Write the conditions for acceptable wave functions. Identify the acceptable wave functions among the following and justify: (i) $x^{4}$ (ii) $\tan \theta$. <br> (b) Show that $\psi=\sin (5 x) \sin (8 y) \sin (2 z)$ is an eigen function of $\nabla^{2}$ and find the eigen value? <br> (c) Predict the value of $\left[\mathrm{x}, \mathrm{p}_{\mathrm{x}}{ }^{2}\right]$ and mention its significance. $(4.5+4+4)$ | K4 | CO3 |


| 9 | (a) Calculate the length of the $\gamma$-carotene molecule which is a conjugated system having 10 double bonds ( +9 single bonds) and a transition wave length of 4300 Å. <br> (b) Write the Hamiltonian and Schrödinger wave equation for hydrogen like atom. Draw the radial plots for $\mathrm{n}=3$ and $1=1$. (5+7.5) | K4 | CO 3 |
| :---: | :---: | :---: | :---: |
| 10 | (a) Explain Born-Oppenheimer approximation and write Kohn-Sham equation. <br> (b) Using the concept of Great Orthogonality theorem and construct $\mathrm{D}_{2 \mathrm{~h}}$ character table. | K4 | CO 3 |
| 11 | (a) What is variation integral? How is it used to determine the energies associated with the trial function $\psi=\mathrm{c}_{1} \psi_{1 \mathrm{a}}+\mathrm{c}_{2} \psi_{1 \mathrm{~b}}$ in the formation of $\mathrm{H}_{2}^{+}$ion? <br> (b) Explain the evaluation of the average energy integrals $\mathrm{H}_{\mathrm{aa}}$ and $\mathrm{H}_{\mathrm{ab}}$. $\quad$ (6.5+6) | K4 | CO 3 |
| SECTION D |  |  |  |
| Answer any ONE of the following |  | x 15 = 15) |  |
| 12 | (a) Derive time dependent Schrödinger wave equation. <br> (b) With the help of perturbation theorem, predict the ground state energy of helium atom. <br>  | K5 | CO 4 |
| 13 | (a) Write the requirement of Hartree-Fock self-consistent field method. <br> (b) Identify the symmetries of IR and Raman vibrational modes of trans-2-butene using the $\mathrm{C}_{2 \mathrm{~h}}$ character table provided. Verify whether this molecule obeys mutual exclusion principle and mention the significance of the Mulliken symbols of modes. (3+12) | K5 | CO4 |
| SECTION E |  |  |  |
| Answer any ONE of the following |  | (1×20=20) |  |
| 14 | (a) Derive the expressions for wave function and energy for a particle in 1-D box of length 1. <br> (b) Obtain the Hamiltonian for simple harmonic oscillator. Prove that the operator $\mathrm{p}_{\mathrm{x}}$ <br> (h/2ri) is <br> Hermitian. <br> (8) <br> (c) The wavenumber of the fundamental vibrational transition of ${ }^{35} \mathrm{Cl}_{2}$ is 564.9 $\mathrm{cm}^{-1}$. Calculate the force constant of the bond (mass of ${ }^{35} \mathrm{Cl}=34.9688 \mathrm{u}$ ). (6) | K6 | CO5 |

a) Apply variation theorem to predict the ground state energy of hydrogen atom using the trial wave function, $\psi=e^{-\alpha r}$.
b) Using Huckel molecular orbital theory, solve the secular determinants for ethylene and allyl radical. Calculate the total $\pi$-electron energy and the stabilisation energy.
c) Discuss the application of direct product principle to verify whether the $\pi \rightarrow \pi^{*}$ transition is allowed in HCHO molecule. The $\mathrm{C}_{2 \mathrm{v}}$ character table is given for reference.

| $C_{2 \mathrm{v}}$ | $E$ | $C_{2}$ | $\sigma_{\mathrm{v}}(x z)$ | $\sigma_{\mathrm{v}}{ }^{\prime}(y z)$ |  |  |
| :--- | ---: | ---: | ---: | :--- | :--- | :--- |
| $A_{1}$ | 1 | 1 | 1 | 1 | $z$ | $x^{2}, y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | -1 | $R_{z}$ | $x y$ |
| $B_{1}$ | 1 | -1 | 1 | -1 | $x, R_{y}$ | $x z$ |
| $B_{2}$ | 1 | -1 | -1 | 1 | $y, R_{x}$ | $y z$ |

