LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2022**

16/17/18UST4AL01 – MATHEMATICAL STATISTICS

Date: 27-06-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

SECTION-A (10 x 2 = 20)

Answer ALL the questions. Each carries 2 marks.

1. Define consistent estimator.
2. Write any two properties of ML estimators.
3. What is the difference between simple hypothesis and alternative hypothesis?
4. Define Power of the test.
5. State the principle of least squares.
6. Mention any two applications of t-distribution in test of significance.
7. What is the difference between parametric and non-parametric tests?
8. Explain Mann Whitney's U test.
9. What is the need for sampling methods?
10. Explain the Multiple linear regression.

SECTION-B (5 x 8 = 40)

Answer any FIVE questions. Each carries 8 marks.

11. (a)Show that the sample mean is an unbiased estimator of the population mean.
    (b) Explain the concept of consistent estimator and also show that in sampling from a N(μ,σ2) population, the sample mean is a consistent estimator of μ.
12. A random sample(X_1, X_2, X_3, X_4, X_5) of size 5 is drawn from normal population with unknown mean μ. Consider the following estimators.
    i) $t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$, ii) $t_2 = \frac{X_1 + X_2}{2} + X_3$ iii) $t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$
    Find λ so that t_3 is an unbiased estimator of μ.Are t_1 and t_2 unbiased? Find the estimator which is best among t_1, t_2 and t_3.
13. State and prove Cramer Rao inequality.
14. State and prove Rao Blackwell theorem.
15. Let p be the probability that a coin will fall head in a single toss in order to test H_0: p=1/2 Vs. H_1 : p=3/4. If the coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.
16. Explain Monotone Likelihood Ratio Property. Find whether the family N (μ,1) satisfied this property.
17. Obtain 100(1-α)% confidence interval for the parameter (i) μ (ii) σ^2 of the Normal distribution.
18. Derive the estimators of regression co efficients in simple linear regression using the method of least squares.

SECTION-C (2 x 20 =40)

Answer any TWO questions. Each carries 20 marks.

19. (a) Describe the procedure of Maximum Likelihood Estimation. (10)

(b) In random sampling from normal population $N(\mu,\sigma^2)$, find the maximum likelihood estimators for i) μ when σ^2 is known ii) σ^2 when μ is known. (10)

20. (a) State and prove Neyman Pearson lemma. (15)

(b) Write the various steps involved in testing of hypotheses. (5)

21. (a) A genetic engineering company claims that it has developed a genetically modified tomato plant that yields on average more tomatoes than other varieties. A farmer wants to test the claim on a small scale before committing to a full-scale planting. Ten genetically modified tomato plants are grown from seeds along with ten other tomato plants. At the season's end, the resulting yields in pound are recorded as below.

Sample 1 (genetically modified)	20	23	27	25	25	25	27	23	24	22
Sample 2 (regular)	21	21	22	18	20	20	18	25	23	20

Test, at the 1% level of significance, whether the data provide sufficient evidence to conclude that the mean yield of the genetically modified variety is greater than that for the standard variety. (10)

(b) The scores on an aptitude test required for entry into a certain job position have a mean of 500 and a standard deviation of 120. If a random sample of 36 applicants has a mean of 546, is there evidence that their mean score is different from the mean that is expected from all applicants? (10)

22. (a) Explain in detail the probability and non probability sampling techniques. (10)

(b) Fit a Simple Linear Regression model to the data given below.

Job Satisfaction measure (Y)	45	35	35	40	55	50	38	55
Supervisor's Score (X_i)	39	40	40	42	45	43	44	47

(10)

##########