LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2022**

PST 2602 – MODERN PROBABILITY THEORY

Date: 24-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions **(10x 2= 20 Marks)**

1. Define measure and probability space.
2. Distinguish between field and σ −field.
3. If $A_n \to A$, then show that $P(A_n) \to P(A)$.
4. Show that $F(x.y) = \begin{cases} 0 & ;\ x,y \le 0,\ x+y \le 1 \\ 1 & ;\ otherwise \end{cases}$ is not a distribution function.
5. If

$$F(x) = \begin{cases} 0 & ;\ x < 0 \\ \frac{1}{2} & ;\ x = 0 \\ \frac{1}{2} + \frac{x}{2} & ;\ 0 < x < 1 \\ 1 & ;\ x \ge 1 \end{cases}$$

Show that F is neither continuous nor discrete.

6. Show that $F_X(x) = \frac{1}{1+e^{-x}}$ is a distribution function.
7. State Liapounov'sform of central limit theorem.
8. Derive Minkowski inequality.
9. Define Bernoulli weak law of large numbers.
10. Show that $\mu[a,b] = F(b) - F(a^-)$, if μ is the Lebesgue Stieltjes measure

PART – B

Answer any FIVE questions **(5x 8= 40 Marks)**

11. State and prove the necessary and sufficient condition for $F(x,y)$ to be a distribution function.
12. Decompose the distribution function

$$F(x) = \begin{cases} 0 & ;\ x < 0 \\ \frac{x+2}{6} & ;\ 0 \le x < 1 \\ \frac{x+3}{6} & ;\ 1 \le x < 2 \\ 1 & ;\ x \ge 2 \end{cases}$$

13. If X and Y are simple random variables, then prove that

$X \ge 0$ a.s. then $E(X) \ge 0$ and X$\ge Y$ a.s. then $E(X) \ge E(Y)$.

14. i) If Z is a complex random variable, then show that $|E(Z)| \le E|Z|$.

ii)If φ is the characteristic function of a general distribution function F, then show that φ is continuous. (4+4)

15. Let P[(X,Y) = (1,1)] = 1/3 = P[(X,Y) = (1,-1)] and P[(X,Y) = (-1,1)] = 1/6 =

P[(X,Y) = (-1,-1)], then find the characteristic function of (X,Y).

16. i) State and prove C_r inequality.

ii) State and prove Basic inequality. (4+4)

17. i) Prove that $X_n \xrightarrow{P} X \Longrightarrow F_n(x) \longrightarrow F(x), x \in C(F)$

ii) Show that $X_n \xrightarrow{r} X \Longrightarrow E|X_n|^r \rightarrow E|X|^r$ (4+4)

18. Find f(x) where $\varphi(t) = \begin{cases} 1-|t|\,; & |t| \leq 1 \\ 0\,; & |t| > 1 \end{cases}$

Part C

Answer any TWO questions **(2x 20= 40 Marks)**

19. i) State and prove Lindberg-Levy central limit theorem.

iii) Show that Liapounov's condition holds then Lindberg Feller condition also holds. (12+8)

20. i)If the pdf of (X, Y) is given by

$$f(x,y) = \begin{cases} 1; & 0 \leq x, y \leq 1 \\ 0\,; & otherwise \end{cases}$$

Obtain the probability distribution of (X+Y).

ii) Let F be a distribution function given by

$$F(x) = \begin{cases} 0 & ; \ x < -1 \\ 1+x & ; \ -1 \leq x < 0 \\ 2+x^2 & ; \ 0 \leq x < 2 \\ 9 & ; \ x \geq 2 \end{cases}$$

If μ is the Lebesgue-Stieltjes measure corresponding to F, compute the measure of $\{x;\ |x| + 2x^2 > 1\}$.

(12+8)

21. i) Let $X_k \sim$ iid as DU{-k, k}, prove that Liapounov's form of central limit theorem holds for $\{X_k\}$.

ii) Find the density of Binomial distribution using inversion formula. (10+10)

22. i) Derive the linearity and scale preserving property of expectation.

ii) Prove the necessary and sufficient condition for convergence in probability and hence show that convergence in probability implies mutual convergence in probability. (8+12)

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