LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

UST 2302 – MATHEMATICAL STATISTICS

(21 BATCH ONLY)

Date: 27-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A				
Answer ALL the Questions				
1.	**Define the following**	**(5 x 1 = 5)**		
a)	Distribution function of a random variable.		K1	CO1
b)	Geometric distribution.		K1	CO1
c)	Gamma distribution.		K1	CO1
d)	t- Distribution.		K1	CO1
e)	Order Statistics.		K1	CO1
2.	**Fill in the blanks**	**(5 x 1 = 5)**		
a)	The range of Pearson's coefficient of correlation is ______________.		K1	CO1
b)	Mean and variance of Poisson distribution are _________________.		K1	CO1
c)	The Moment generating function of the normal distribution is _________.		K1	CO1
d)	The test statistic for t is______________.		K1	CO1
e)	The sample variance is ______________.		K1	CO1
3.	**Match the following**	**(5 x 1 = 5)**		
a)	If X and Y are independent if and only if Cov(X,Y)	$\frac{1}{\theta}$ *and* $\frac{1}{\theta^2}$	K2	CO1
b)	Hypergeometric Distribution	Continuous	K2	CO1
c)	Mean and Variance of exponential distribution are	0	K2	CO1
d)	Gamma Distribution	Discrete	K2	CO1
e)	Uniform Distribution	$f(x)=\frac{1}{b-a}$; $a\le x\le b$	K2	CO1
4.	**TRUE or FALSE**	**(5 x 1 = 5)**		
a)	In probability, a real-valued function, defined over the sample space of a random experiment, is called a **random variable.**		K2	CO1
b)	The mean of Hypergeometric distribution is $\frac{n}{N}$.		K2	CO1
c)	In Gamma distribution, mean and variance are different.		K2	CO1
d)	The d.f for related or paired sample t test is n-1.		K2	CO1
e)	F test is used to test for equality of variances from two normal populations.		K2	CO1

SECTION B

Answer any TWO of the following in 100 words			(2 x 10 = 20)
5.	Calculate coefficient of correlation from the following data. X: 45, 55, 56, 58, 60, 65, 68, 70, 75, 80, 85 Y: 56, 50, 48, 60, 62, 64, 65, 70, 74, 82, 90	K3	CO2
6.	A random variable x has the following probability distribution x: -2, -1, 0, 1, 2, 3 P(x): 0.1, k, 0.2, 2k, 0.3, 3k a) Compute the value of k b) Compute $P(x<2)$ c) Compute $P(-2<x<2)$	K3	CO2
7.	Show that, Mean= $\frac{q}{p}$ and variance = $\frac{q}{p^2}$	K3	CO2
8.	Demonstrate Chi square distribution and point out its applications and find its MGF.	K3	CO2

X	45	55	56	58	60	65	68	70	75	80	85
Y	56	50	48	60	62	64	65	70	74	82	90

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

SECTION C

Answer any TWO of the following in 100 words			(2 x 10 = 20)
9.	State and prove Chebyshev's inequality.	K4	CO3
10.	A manufacturer of pins knows that, 2% of the products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. a) What is the probability that a box will fail to that guaranteed quantity? b) Compute $P(x=0)$ and $P(x\leq 2)$	K4	CO3
11.	Point out the moments of beta distribution of first kind and hence find its mean and variance.	K4	CO3
12.	a) Explain F distribution and give the F-test statistic. (4) b) The mean weekly sales of soap bars in departmental stores was 146·3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153·7 and showed a standard deviation of 17·2. Was the advertising campaign successful? (6)	K4	CO3

SECTION D

Answer any ONE of the following in 250 words			(1 x 20 = 20)
13.	Two random variables X and Y have the following joint probability density function, $f(x,y) = f(x) = \begin{cases} 2-x-y, & 0 \leq x < 1 \\ & 0 \leq y < 1 \\ 0, & otherwise \end{cases}$ Infer the results of, (i) The marginal probability density function of X and Y (ii) Conditional density function of X and Y (iii) Variance of X and Y (iv) Covariance between X and Y	K5	CO4
14.	a) Let X be normally distributed with mean 8 and standard deviation 4. Evaluate: i) $P(5\leq X\leq 10)$ ii) $P(X\leq 5)$ iii) $P(X\geq 15)$ (12) b) Derive the probability density function of Student's t distribution. (8)	K5	CO4

SECTION E

Answer any ONE of the following in 250 words			**(1 x 20 = 20)**
15.	State and prove the central limit theorem.	K6	CO5
16.	a) Derive the mgf of Poisson distribution and hence find its mean and variance. (10) b) The table given below shows the data obtained during outbreak of small pox. (10) <table><tr><td></td><td>Attacked</td><td>Not Attacked</td></tr><tr><td>Vaccinated</td><td>31</td><td>469</td></tr><tr><td>Not Vaccinated</td><td>185</td><td>1315</td></tr></table>Test the effectiveness of vaccination in preventing attack from small pox. Test at 5 % level of significance.	K6	CO5

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