LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2022**

UST 2501 – CONTINUOUS DISTRIBUTIONS

Date: 16-06-2022 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART A

Answer ALL the questions **(10X2=20)**

1. Define pdf of a continuous random variable X.
2. Write any two properties of a distribution function.
3. If $f(x,y) = \begin{cases} 2-x-y\,; 0 \le x \le 1,\ 0 \le y \le 1 \\ \quad 0 \quad ; \quad otherwise \end{cases}$
   Find the marginal density of X.
4. Define continuous uniform distribution.
5. If $X \sim N(\mu, \sigma^2)$, then write the pdf of $= \frac{X-\mu}{\sigma}$.
6. Find the MGF of Gamma distribution.
7. Find the distribution function of exponential distribution with parameter θ.
8. If $f(x) = 6x(x-1); 0 \le x \le 1$, check whether f(x) is a pdf.
9. Write the density function of F distribution with (n_1, n_2) degrees of freedom.
10. Find the cdf of the smallest order statistic $X_{(1)}$.

PART B

Answer any FIVE questions **(5X8=40)**

11. If X_1 and X_2 are independent rectangular variates on [0,1], find the distribution of $\frac{X_1}{X_2}$.
12. Derive the MGF of a normal distribution and hence prove that a linear combination of independent normal variates is also a normal variate.
13. i) Define bivariate normal distribution.
    ii) Let $X_1, X_2 \sim$ bivariate normal distribution, the show that X_1 is independent of X_2 if and only if $\rho = 0$. (4+4)
14. Find the rth moment of Beta distribution of first kind and hence find its mean and variance.
15. Find the pdf of a single order statistic $X_{(r)}$.
16. Let $X \sim N(0,1)$ and $Y \sim \chi^2_{(n)}$, X is independent of Y then find the distribution of $\frac{X}{\sqrt{Y/n}}$.
17. Prove that V(X) = E[V(X|Y)] + V[E(X|Y)].
18. i)Find the mean deviation about mean for a normal distribution.

ii) If $f(x,y) = \begin{cases} 6x^2y & ; \ 0 < x < 1, 0 < y < 1 \\ 0 \ ; & otherwise \end{cases}$

verify that $\int_0^1 \int_0^1 f(x,y)dxdy = 1$. (4+4)

PART C

Answer any TWO questions **(2X20=40)**

19. i) If X and Y are independent with a common pdf $f(x) = \begin{cases} e^{-x}; x \geq 0 \\ 0 \quad ; x < 0 \end{cases}$

Find the pdf of X – Y.

ii) State and prove the lack of memory property of exponential distribution. (15+5)

20. i) Let X have a standard Cauchy distribution, find pdf for X^2 and identify its distribution.

ii) Let $X \sim N(0,1)$ and $Y \sim N(0,1)$ be independent random variable, find the distribution of $\frac{X}{Y}$ and identify it. (8+12)

21. i) Find the joint pdf of order statistics $X_{(r)}$ and $X_{(s)}$.

ii) Find the pdf of rth order statistics of exponential distribution. (13+7)

22. State and prove Lindberg Levy central limit theorem.

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