LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2023**

PST1MC01 – ADVANCED DISTRIBUTION THEORY

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A			
Answer ALL the questions			
1	**Answer the following /Definitions** **(5 x 1 = 5)**		
a)	Write the pdf of truncated Poisson truncated at '0' and obtain its moment generating function.	K1	CO1
b)	State the properties of a distribution function.	K1	CO1
c)	Let (X_1, X_2) ~BVE, prove that X_1 and X_2 are independent if and only if $\lambda_{12}=0$.	K1	CO1
d)	Let $(X_1, X_2) \sim$ BVP $(\lambda_1, \lambda_2, \lambda_{12})$. Find its probability generating function.	K1	CO1
e)	If X is Inverse Gaussian, show that 2X is also Inverse Gaussian.	K1	CO1
2	**True or False/ Answer the following / Definitions** **(5 x 1 = 5)**		
a)	Let X_1, X_2, X_3 be iid $N(\mu, \sigma^2)$ random variables. Then, $2X_1+3X_2-X_3$ and $3X_1-X_2+3X_3$ are independent. **(TRUE/FALSE)**	K2	CO1
b)	Write the joint pdf of i^{th} and j^{th} order statistics.	K2	CO1
c)	If X follows log-normal then prove that 1/X is also log normal.	K2	CO1
d)	Write any one difference between central and non-central distributions.	K2	CO1
e)	What are compound distributions?	K2	CO1
SECTION B			
	Answer any THREE of the following **(3 x 10 = 30)**		
3	Show that the Binomial distribution is a power series distribution. Obtain the MGF of a power series distribution. Also Obtain the recurrence relation for the cumulants of power series distributions.	K3	CO2
4	State and establish Skitovitch theorem regarding Normal distributions.	K3	CO2
5	Let X1,X2 be independent and identically distributed non negative, integer valued random variables Then show that X_1 is geometric if and only if $Min\{X_1, X_2\}$ is geometric.	K3	CO2
6	Derive the probability density function of non-central F-distribution.	K3	CO2
7	Let $X_1, X_2 \ldots X_n$ be iid $N(\mu, \sigma^2)$ random variables . Using the concept of Quadratic forms, show that the sample mean and the sample variance are independent.	K3	CO2
	SECTION C		
Answer any TWO of the following **(2 x 12.5 = 25)**			
8	a) Establish the pdf of Trinomial distribution. **(5)** b) Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$ then obtain the mean and variance of X_2. **(7.5)**	K4	CO3
9	Let $X_1, X_2 \ldots X_n$ be a random sample from $f(x) = \alpha e^{-\alpha x}, x > 0, \alpha > 0$. Let $D_{ik} = (n-k+1)[X_{(k)} - X_{(k-1)}]$ where $X_{(k)}$ denotes the k^{th} order statistic. Show that $D_1, D_2 \ldots D_n$ are iid exponential random variables.	K4	CO3
10	Evaluate the moment generating function of non-central chi square distribution.	K4	CO3

11	Let $X_1, X_2 \ldots X_n$ be independent and identically distributed Normal variables with mean zero and the variance σ^2. Show that $X'AX / \sigma^2$ is distributed as chi-square if A is idempotent.	K4	CO3

	SECTION D		
	Answer any ONE of the following **(1 x 15 = 15)**		
12	State and prove Cochran's theorem.	K5	CO4
13	If (X_1,X_2) has Bivariate Normal distribution then obtain the Marginal distribution of X_1 and the conditional distribution of X_1 given $X_2= x_2$.	K5	CO4
	SECTION E		
	Answer any ONE of the following **(1 x 20 = 20)**		
14	a) If (X_1, X_2) has Bivariate Poisson distribution then obtain the correlation between X_1 and X_2. **(10)** b) Evaluate the Mean and variance of truncated exponential distribution truncated at 5. **(10)**	K6	CO5
15	a) The distribution function of random variable X is given by, $$F(x) = \begin{cases} 0\,; & x < -1 \\ \dfrac{x+2}{4} - 1; & -1 \le x < 1 \\ 1\,; & 1 \le x < \infty \end{cases}$$ Decompose the distribution function. Find the mean and variance. Justify your Answer. **(10)** b) Let $X\vert\theta \sim B(n, \theta)$ and, $\theta \sim Beta_1(\alpha, \beta)$, $where\ \alpha, \beta$ are known . Find the compound distribution and Mean. **(10)**	K6	CO5

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