LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

17/18UST3AL01 – MATHEMATICAL STATISTICS - I

Date: 01-12-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION - A

Answer ALL **(10 × 2 =20)**

1. Define random variable and give an example.
2. State and Prove Addition Theorem of Probability.
3. A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?
4. How do you derive probability density function from the probability distribution function?
5. State the assumptions of Binomial distribution.
6. Derive the mean of Poisson distribution.
7. Write the moment Generating function of two parameter Gamma distribution.
8. Write any four characteristics of Normal distribution.
9. Define Chi- square statistic.
10. What is convergence in distribution?

SECTION – B

Answer Any FIVE from the following **(5 × 8 = 40)**

11. (a) Define : Conditional Probability

    (b) State and prove Bayes' theorem. **(2+6)**
12. State and prove Chebyshev's inequality.
13. Prove that for Poisson distribution Mean is equal to variance.
14. Prove that Exponential distribution lacks memory.
15. Derive the moment Generating function of χ^2 distribution.
16. Obtain the Mean and variance of Gamma distribution.
17. Derive the probability density functions of $X_{(1)}$ and of $X_{(n)}$.
18. Calculate Karl Pearson's correlation coefficient between height (in inches) and weight (in kg) from the data given below.

Height:	60	63	65	64	68
Weight:	50	53	60	67	70

SECTION – C

Answer any TWO from the following **(2 × 20 = 40)**

19. The joint density function of two continuous random variables X and Y is

$$f(x,y) = \begin{cases} cxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

(b) Find P(X≥ 3, Y≤ 2).

(c) Find P(1< X < 2, 2 < Y < 3).

(d) Find E(X) and Var(X). **(5+5+5+5)**

20. (a) Prove that a linear combination of independent normal random variables is also a normal random variate.

(b) Derive the mean and variance of Beta distribution of first kind. **(10+10)**

21. Derive the probability density of F distribution and derive its mean and variance.

22. State and prove the Central Limit Theorem.

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