LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2022**

PST1MC01 – ADVANCED DISTRIBUTION THEORY

Date: 23-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A		
	Answer ALL the Questions		
1	**Answer the following / Definitions** **(5 x 1 = 5)**		
a)	Write the pdf of truncated Binomial distribution truncated at ‘0’ and obtain its moment generating function.	K1	CO1
b)	State the difference between distribution function and its probability density function.	K1	CO1
c)	Let $X_1, X_2 \ldots X_n$ be a random sample from Geometric distribution. Show that first order statistic also has Geometric distribution.	K1	CO1
d)	Define non-central F distribution.	K1	CO1
e)	Define a Quadratic form in n variables.	K1	CO1
2	**Answer the following / MCQ/Definitions** **(5 x 1 = 5)**		
a)	Show that Geometric distribution satisfies Lack of memory property.	K2	CO1
b)	Let $X_1, X_2,\ldots X_n$ be iid inverse Gaussian random variables, Then prove that the arithmetic mean of $X_1, X_2,\ldots X_n$ is also Inverse Gaussian distribution.	K2	CO1
c)	Let Z = (X, Y) be a Bivariate Normal random variable. Then, which of the following statements is false? (a) X and Y are independent if and only if they are uncorrelated. (b) X + Y is univariate normal. (c) Y | X = x is distributed as a Normal random variable. (d) X + Y and X − Y are independent.	K2	CO1
d)	Write the moment generating function of non central chi square distribution.	K2	CO1
e)	If X follows log-normal then prove that 1/X is also log normal.	K2	CO1
	SECTION B		
	Answer any THREE of the following in 500 words **(3 x 10 = 30)**		
3	The distribution function of random variable X is given by, $$F(x)=\begin{cases}0; & x<2\\ \frac{2}{3}x-1; & 2\le x<3\\ 1; & 3\le x\end{cases}$$ Decompose the distribution function. Find the mean and variance.	K3	CO2
4	Let X be a non-negative absolutely continuous random variable, Then X obeys lack of memory property if and only if X is exponential.	K3	CO2
5	Show that mean>median>mode for lognormal distribution.	K3	CO2
6	Derive the pdf of non-central t-distribution.	K3	CO2
7	(i) State any two differences between central distributions and non-central distributions. **(5)** (ii) Explain the importance of Compound distributions. **(5)**	K3	CO2
	SECTION C		
	Answer any TWO of the following **(2 x 12.5 = 25)**		
8	Let X follow the power series distribution. Obtain the recurrence relationship for cumulants and hence obtain the mean and variance of Log series distribution.	K4	CO3
9	Let X_1, X_2, X_3, X_4 be iid N(0,1) random variables. Find the distribution of (i) X_1X_2	K4	CO3

	(ii) $X_1X_2-X_3X_4$		
10	a) Establish the pdf of Bivariate binomial distribution. **(5)** b) Let $(X_1, X_2) \sim BB(n, p_1,p_2,p_{12})$ then obtain the mean and variance of X_1. **(7.5)**	K4	CO3
11	Let $X\|\theta \sim N(\theta, 1)$ and θ =0.1, 0.2, 0.3and $\theta \sim DU\{0.1, 0.2, 0.3\}$ known. Find the compound distribution and its Mean.	K4	CO3

	SECTION D		
	Answer any ONE of the following **(1 x 15 = 15)**		
12	Let $X_1,X_2,\ldots X_n$ be iid Exponential random variables with parameter λ. Prove that a) $nX_{(1)}$ is exponential with parameter λ. b) $\sum_{i=2}^{n}(X_{(i)} - X_{(1)}) \sim G(\lambda, n-1)$. c) $X_{(1)}$ is independent of $\sum_{i=2}^{n}(X_{(i)} - X_{(1)})$. **(5+5+5)**	K5	CO4
13	Let X follows Inverse Gaussian with parameters (μ ,σ^2). Establish the r^{th} cumulants of the inverse Gaussian Distribution and hence obtain the mean and variance.	K5	CO4
	SECTION E		
	Answer any ONE of the following **(1 x 20 = 20)**		
14	a) Examine whether the geometric mean of 'n' independent and identically log normal random variables follow log normal. **(10)** b) Let (X_1, X_2) have Bivariate Binomial with parameters n, p_1, p_2 and p_{12}. Verify whether X1 given $X_2=x_2$ is equal in distribution to $U_1+ V_1$ where U_1 and V_1 are independent. Hence Obtain the Correlation Coefficient between X_1 and X_2. **(10)**	K6	CO5
15	a) Let $Q_1,Q_2,\ldots Q_k$ be k quadratic forms in iid $N(0 ,\sigma^2)$ random variables $X_1,X_2,\ldots X_n$. Let $X'X = Q_1+Q_2+\ldots+Q_k$ and $\rho(A_j) = r_j, j = 1,2,\ldots,k$ Then $\sum_{i=1}^{k} r_j = n$ if and only if (i) $Q_1,Q_2,\ldots Q_k$ are independent. (ii) $\frac{Q_i}{\sigma^2} \sim chi\ square\ with\ r_j\ df\ j = 1,2,\ldots,k$. **(10)** b) Let X_1, X_2 be iid $N(0, \sigma^2)$ random variables. Examine whether i. X_1+X_2 and $(X_1-X_2)^2$ are independent. ii. $(X_1+X_2)^2$ and X_1^2-X_2^2 are independent. **(10)**	K6	CO5

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