# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2022**

**PST1MC03 – STATISTICAL MATHEMATICS**

Date: 28-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

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| **SECTION A** | | | |
| **Answer ALL the questions** | | | |
| **1** | **Define the following.** **(5 x 1 = 5)** | | |
| a) | Raabe's test | K1 | CO1 |
| b) | Maxima and minima of a function | K1 | CO1 |
| c) | Upper and lower Riemann integrals | K1 | CO1 |
| d) | Basis and Dimension | K1 | CO1 |
| e) | Inner product space | K1 | CO1 |
| 2 | **Fill in the blanks.** **(5 x 1 = 5)** | | |
| a) | The function $f(x)=\|x+2\|$ is not differentiable at the point ____________ | K2 | CO2 |
| b) | $\lim_{n\to\infty}\frac{1}{n}(1+2^{1/2}+3^{1/3}+\ldots+n^{1/n})$ is equal to _______________. | K2 | CO1 |
| c) | Let $f(x)=k$ (= constant) on [a,b] and g be monotonically, non-decreasing on [a,b]. Then $\int_a^b f\ dg=$ ________. | K2 | CO1 |
| d) | Any subset containing (n+1) vectors of an n-dimensional vector space is linearly __________ | K2 | CO1 |
| e) | A finite dimensional real inner product space is called as _______________ | K2 | CO1 |
| **SECTION B** | | | |
| | **Answer any THREE of the following questions.** **(3 x 10 = 30)** | | |
| 3 | Prove that the sequence $\{u_n\}$ defined by $u_1=\sqrt{7}$, $u_{n+1}=\sqrt{7+u_n}$ converges to the positive root of the equation $x^2-x-7=0$. | K3 | CO2 |
| 4 | A function $f(x)$ is defined as follows: $f(x)=\begin{cases}1+\sin x & if\ \ 0<x<\frac{\pi}{2}\\ 2+\left(x-\frac{\pi}{2}\right)^2 & if\ \ x\geq\frac{\pi}{2}\end{cases}$. Examine its continuity and derivability at $x=\frac{\pi}{2}$. | K3 | CO2 |
| 5 | If S, T are two subsets of a vector space V, then prove that (i) $S\subseteq T\Rightarrow L(S)\subseteq L(T)$ (ii) $L(SUT)=L(S)+L(T)$ (iii) $L[L(S)]=L(S)$. | K3 | CO2 |
| 6 | (i).If $f\in R[a,b]$, then prove that $m(b-a)\leq\int_a^b f(x)\,dx\leq M(b-a)\ if\ \ b\geq a$; where m and M are the infimum and supremum of $f$ on [a,b]. (ii) Let $f(x)=x$ for $x\in[0,1]$ and let $P=\left\{0,\frac{1}{3},\frac{2}{3},1\right\}$ be a partition of [0,1]. Compute U(P,f) and L(P,f). | K3 | CO2 |
| 7 | Find the characteristic and minimal polynomials of each of the following matrices | K3 | CO2 |

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| | $A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & -2 \\ -1 & -1 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 2 & -1 \\ 3 & 8 & -3 \\ 3 & 6 & -1 \end{bmatrix}$ | | |
| | **SECTION C** | | |
| | **Answer any TWO of the following questions. (2 x 12.5 = 25)** | | |
| 8 | (i).Prove that a sequence is convergent if and only if its a Cauchy sequence. **(7+5.5)**<br>(ii). Show by applying Cauchy's convergence criterion that the sequence $\{a_n\}$ where $a_n = 1+\frac{1}{3}+\frac{1}{5}+...+\frac{1}{2n-1}$ is not convergent. | K4 | CO3 |
| 9 | Let $f: R \to R$ be such that $f(x)=\begin{cases} \frac{\sin\,(a+1)\,x+\sin\,x}{x} & for\;\; x<0 \\ c & for\;\; x=0 \\ \frac{(x+bx^2)^{1/2}-x^{1/2}}{b\,x^{3/2}} & for\;\; x<0 \end{cases}$. Determine the values of a, b, c for which the function is continuous at x = 0. | K4 | CO3 |
| 10 | Prove that the set of all ordered n-tuples forms a vector space over a field F. | K4 | CO3 |
| 11 | Determine the characteristic roots and the corresponding characteristic vectors of the matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ and show that the matrix satisfies Cayley Hamilton theorem. | K4 | CO3 |

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| | **SECTION D** | | |
| | **Answer any ONE of the following questions (1 x 15 = 15)** | | |
| 12 | Prove that a monotonic sequence is never oscillatory. | K5 | CO4 |
| 13 | (i).State and prove L Hospital Rule for determining the true value of the indeterminate form $0/0$. **(7+8)**<br>(ii) Evaluate: (a) $\underset{x\to 0}{Lt}\frac{e^x\sin x-x-x^2}{x^2+x\log(1-x)}$, (b). $\underset{x\to 0}{Lt}\; x^m(\log x)^n$. | K5 | CO4 |
| | **SECTION E** | | |
| | **Answer any ONE of the following questions (1 x 20 = 20)** | | |
| 14 | (i).Using integral test, test the convergence of the series**:** **(10+10)**<br>$\sum_{n=3}^{\infty}\frac{1}{n\,\log n\,(\log\,\log n)^p},\, p>0$<br>(ii) Reduce the following quadratic forms in three variables to real canonical form and find its rank and signature:<br>$6x_1^2+3x_2^2+14x_3^2+4x_2x_3+18x_3x_1+4x_1x_2$. | K6 | CO5 |
| 15 | (i).Given that the set S below spans $R^3$. Find a basis of $R^3$ which is contained in S.<br>$\{(2,6,-3),(5,15,-8),(3,9,-5),(1,3,-2),(5,3,-2)\}$ (10+10)<br>(ii) Let f and g be two functions. If $\lim_{x\to a}f(x)=l$ and $\lim_{x\to a}g(x)=m$, then prove that<br>(a) $\lim_{x\to a}[f(x).g(x)]=lm$ (b) $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{l}{m};m\neq 0$ | K6 | CO5 |

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