LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2022**

PST 3502 – STOCHASTIC PROCESSES

Date: 25-11-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

SECTION – A

Answer ALL the Questions. **10 x 2 = 20 Marks**

1. Define Markov process.
2. State any two properties of periodicity.
3. Show that recurrence is a class property.
4. Write a note on absorption probability in a Markov chain.
5. Write the postulates for a pure birth process.
6. Cite any two examples for renewal process.
7. When a process is called (i) Supermartingale and (ii) Submartingale ?
8. Let the Markov chain have the state space S = {1,2 ,3} with the following one-step transition probabilities P_{11} =1/2 , P_{12} =1/4 , P_{13} =1/4 , P_{21} =1/3 , P_{23} =2/3 , P_{31} =1/2 and P_{32} =1/2. If $P(X_1 = 1) = P(X_1 = 2) = 1/8$, find $P(X_1 = 3 , X_2 = 2, X_3 = 1)$.
9. Explain branching process with an example.
10. Illustrate Stationary process.

SECTION – B

Answer any FIVE Questions. **5 x 8 = 40 Marks**

11. Explain (i) One-dimensional random walk and (ii) Spatially homogeneous Markov chains.

12. (a) Show that communication is an equivalence relation. **(3)**
    (b) State and prove the necessary and sufficient condition for a state of a Markov chain is recurrent. **(5)**

13. If the Markov chain has the states 1,2 and 3 with the one-step transition probabilities : $P_{12} = 2/3$, $P_{13} = 1/3$, $P_{21} = P_{23} = 1/2$, $P_{31} = P_{32} = 1/2$, find stationary distribution.

14. Derive $P_n(t)$ for Poisson process by clearly stating the postulates.

15. Explain (i) Right regular sequences and induced martingales for Markov chains and (ii) Doob's Martingale process. **(5 + 3)**

16. Narrate Type I counter model in renewal process.

17. Explain branching process with the help of any four probability generating functions.

18. Show that the moving average process is covariance stationary.

SECTION - C

Answer any TWO Questions. **2 x 20 = 40 Marks**

19.(a) Show that two-dimensional random walk is recurrent. **(5)**

(b) Prove that three-dimensional random walk is transient. **(15)**

20. (a) State and prove the basic limit theorem of Markov chains. **(12)**

(b) State and prove the theorem used to find stationary probability distribution when the Markov chain is positive recurrent, irreducible and aperiodic. **(8)**

21.(a) Derive the backward and forward Kolmogorov differential equations for birth and death process. **(12)**

(b) State and prove the elementary renewal theorem. **(8)**

22. Establish the probability generating function relations for branching process and hence derive its mean and variance .

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