LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **NOVEMBER 2022**

UST 2501 – CONTINUOUS DISTRIBUTIONS

Date: 29-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Q. No	
	PART – A
	Answer ALL the questions **(10 X 2 = 20)**
1	Define joint density function.
2	State the condition under which gamma distribution tends to normal distribution.
3	If X is uniformly distributed with mean 1 and variance 4/3, find $P(X<0)$.
4	What is meant by stochastic independence?
5	State additive property of gamma distribution.
6	Let X be a random variable, then $f(x)=\begin{cases} ke^{-2x}; & x\geq 0 \\ 0 \quad ; & elsewhere \end{cases}$ to be density function. Find the value of k ______.
7	State any two applications of chi-square distribution.
8	Differentiate between normal and standard normal distribution.
9	Write the mean and variance of t-distribution.
10	Define order statistics.
	PART – B
	Answer any FIVE questions **(5 X 8 = 40)**
11	A r.v X is distributed at random between the values 0 and 4 and its p.d.f is given by: $f(x,y)=k\,x^3\,(4-x)^2$. Calculate (i) the value of k (ii) mean and variance (iii) standard deviation.
12	(i) Explain the procedures for generating random numbers in uniform distribution. (ii) Calculate a student randomly draws the following four uniform (0,1) numbers are 0.3, 0.5, 0.6, 0.8. Use the four uniform (0,1) numbers to generate three random numbers that follow an uniform distribution with parameters a=40 and b=50.
13	If the random variables X_1 and X_2 are independent and follow chi-square distribution with n d.f., show that $\sqrt{n}(X_1-X_2)/2\sqrt{X_1X_2}$ is distributed as Student's t with n d.f., independently of X_1+X_2.
14	Prove that a limiting form of binomial distribution tends to normal distribution.
15	In an examination it is laid down that a student passes if he secures 30 percent or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 10% of the students failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division.
16	Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population with mean μ and variance σ^2.

	Then prove that $\sum_{i=1}^{n}\left(\frac{X_i-\overline{X}}{\sigma}\right)^2$ is a χ^2 variate with (n-1) d.f.
17	Derive the m.g.f of gamma distribution and hence find its mean and variance.
18	Explain the joint p.d.f k^{th} order statistics.
PART – C	
Answer any TWO questions	**(2 X 20 = 40)**
19	Two random variables X and Y have the following joint probability density function: $f(x,y)=\begin{cases}(2-x-y); & 0\le x\le 1\ ;\ 0\le y\le 1\\ 0 & ; \quad elsewhere\end{cases}$. Compute (i) Marginal density functions of X and Y, (ii) Conditional density functions of X given Y=y and Y given X=x (iii) E(X) and E(Y) (iv) Var(X) and Var(Y) (v) Covariance between X and Y.
20	State and prove central limit theorem.
21	(i).Derive the moments of beta distribution of second kind and hence find its mean and variance. (ii). Prove that exponential distribution has a lack of memory property. **(10+10)**
22	Derive the moments of t-distribution and hence find its β_1 and β_2

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