LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2022**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 03-12-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL the Questions

1.	**Define the following**		**(5 x 1 = 5)**
a)	Symmetric and Skew-symmetric matrix	K1	CO1
b)	Addition of matrices	K1	CO1
c)	Subspace of a vector space	K1	CO1
d)	Characteristic equation	K1	CO1
e)	Positive definite quadratic form	K1	CO1
2.	**Fill in the blanks**		**(5 x 1 = 5)**
a)	A matrix **A** such that $\mathbf{A}^2 = \mathbf{A}$ is called ____________	K1	CO1
b)	If two rows (or two columns) of a matrix are identical, the value of the determinate is ___________	K1	CO1
c)	Any infinite set of vectors of V is linearly independent if its every finite subset is linearly ___________	K1	CO1
d)	The characteristic roots of a real symmetric matrix are ______________	K1	CO1
e)	A real symmetric matrix A is said to be positive definite if the quadratic form X^TAX is_____________	K1	CO1
3.	**True or False**		**(5 x 1 = 5)**
a)	Any $1\times n$ matrix which has only one row and n columns is called a column vector.	K2	CO1
b)	If all the elements of a row (or a column) of a matrix are zero, the value of the determinant is non zero.	K2	CO1
c)	The set $W = \{(a,0,b): a,b \in R\}$ is a subspace of $R^3(R)$.	K2	CO1
d)	The characteristic roots of an orthogonal matrix are of unit modulus.	K2	CO1
e)	A real symmetric matrix is positive definite if and only if all its eigen values are positive.	K2	CO1
4.	**Match the following**		**(5 x 1 = 5)**
a)	$\sum_{i=1}^{n} a_{ii}$ — Quadratic form	K2	CO1
b)	The method of solving n equations in n unknowns — Three	K2	CO1
c)	The dimension of a vector space $R^3(R)$ is — A	K2	CO1
d)	$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j$ — Trace of a matrix	K2	CO1
e)	$(A^\theta)^\theta$ — Cramer's rule	K2	CO1

SECTION B

Answer any TWO of the following questions			**(2 x 10 = 20)**
5.	Evaluate $\Delta = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$.	K3	CO2
6.	Prove that the equations x+y+z=-3, 3x+y-2z=-2, 2x+4y+7z=7 are not	K3	CO2

	consistent.		
7.	Show that, if A be any n-rowed square matrix, then (Adj A) A = A (Adj A) = $\|A\| I_n$, where I_n is the n-rowed unit matrix.	K3	CO2
8.	Explain the elementary properties of a vector space.	K3	CO2

SECTION C

Answer any TWO of the following questions (2 x 10 = 20)

9.	Explain the properties of matrix multiplication.	K4	CO3
10.	Reduce the matrix $A=\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ to canonical form.	K4	CO3
11.	Show that the set {(1,2,1,0), (3,-4,5,6), (2,-1,3,3), (-2,6,-4,-6)} of $V_4(R)$ is linearly dependent.	K4	CO3
12.	Determine a non-singular matrix P such that P^TAP is a diagonal matrix, where $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$.	K4	CO3

SECTION D

Answer any ONE of the following question (1 x 20 = 20)

13.	Write down in matrix form the system of linear equations 2x-y+3z=9, x+y+z=6, x-y+z=2 and find A^{-1} and hence solve the given equations by using inverse of a matrix.	K5	CO4
14.	(i) Determine the rank of the following matrix $A=\begin{bmatrix} -2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (10) (ii) Write the polynomial $f(x)=x^2+4x-3$ over R as a linear combination of the polynomials $f_1(x)=x^2-2x+5$, $f_2(x)=2x^2-3x$ and $f_3(x)=x+3$. (10)	K5	CO4

SECTION E

Answer any ONE of the following question (1 x 20 = 20)

15.	Reduce the following quadratic form to canonical form and find its rank and signature $6x_1^2+3x_2^2+14x_3^2+4x_2x_3+18x_3x_1+4x_1x_2$.	K6	CO5
16.	(i) Obtain the characteristic equation of the matrix $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify Cayley-Hamilton theorem for this matrix. (12+8) (ii) Justify that the matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is unitary.	K6	CO5

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