# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

SIXTH SEMESTER – **NOVEMBER 2022**

**UST 6502 – OPERATIONS RESEARCH**

Date: 02-12-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION - A

Answer ALL questions. (10 x 2 = 20 marks)

1. What is an 'Artificial Variable' and why is it necessary to introduce it?
2. Define Slack and Surplus variables.
3. In an LPP with some 'unrestricted variables', what changes are made in order to ensure non-negativity requirements?
4. Define 'primal problem' when an LPP is written in 'symmetrical' primal-dual form.
5. Define a LOOP in Transportation Table and give an example.
6. What is meant by 'unbalanced transportation problem'?
7. Explain 'assignment problem' with an example.
8. Write the formulae for finding $p_1$, $p_2$, $q_1$, $q_2$, and $\nu$ for 2 x 2 games without saddle points.
9. Find the minimax and maximin values for the following payoff matrix:
$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}.$$
10. Define a Critical Path, stating the conditions for a critical activity (i, j).

SECTION – B

Answer any FIVE questions. (5 x 8 = 40 marks)

11. A company sells two different products A and B. The company makes a profit of Rs.40 and Rs.30 per unit on products A and B respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 30,000 man hours. It takes three hours to produce one unit of A and one hour to produce one unit of B. The market has been surveyed, and the company officials feel that the maximum number of units of A that can be sold is 8,000 and the maximum of B is 12,000 units. Formulate this problem as an L.P. P.
12. Show that the following LPP has alternative optima:
    Maximize $z = 2x_1 + 4x_2$ subject to the constraints:
    $x_1 + 2x_2 \le 5$, $x_1 + x_2 \le 4$; and $x_1, x_2 \ge 0$.
13. Find all the basic solutions to the system of linear equations:
$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$
    Are the solutions degenerate?
14. Discuss the relationship between primal LPP and its dual LPP.
15. Verify that the dual of dual is primal for the following L.P.P.:
    Maximize $z = 8x_1 + 3x_2$
    subject to the constraints:
$$x_1 - 6x_2 \le 2$$
$$5x_1 + 7x_2 = -4$$
$$x_1, x_2 \ge 0.$$
16. State a procedure of drawing minimum number of lines to cover all the zeroes (0's) of a reduced matrix in Assignment Problem.

17. Construct the network diagram comprising activities A, B, …, H and I such that the following constraints are satisfied:
$A < B$; $B < E, J$; $C < G$; $D < C, F, A$; $E, J < I$; $F < H$; $G < B$; $H < B$.
The notation $X < Y$ means that the activity X must be finished before Y can begin.

18. Solve graphically the game whose payoff matrix is
$$\begin{bmatrix} 2 & 3 & 11 \\ 7 & 5 & 2 \end{bmatrix}.$$

SECTION – C

Answer any TWO questions. (2 x 20 = 40 marks)

19(a) Use Big-M method to solve the following L.P.P.:
Maximize $z = 3x_1 + 2x_2 + 3x_3$
subject to the constraints:
$2x_1 + x_2 + x_3 \le 2$, $3x_1 + 4x_2 + 2x_3 \ge 8$, $x_1, x_2, x_3 \ge 0$. (10)

(b) Use two-phase simplex method to maximize $z = 3x_1 + 2x_2$ subject to the constraints: $2x_1 + x_2 \le 2$, $3x_1 + 4x_2 \ge 12$, $x_1, x_2 \ge 0$. (10)

20. Determine an initial feasible solution to the following transportation problem using the north west corner rule. Also determine the optimum solution by MODI method.

| Origin | Destination $D_1$ | $D_2$ | $D_3$ | $D_4$ | Availability |
|---|---|---|---|---|---|
| $O_1$ | 6 | 4 | 1 | 5 | 14 |
| $O_2$ | 8 | 9 | 2 | 7 | 16 |
| $O_3$ | 4 | 3 | 6 | 2 | 5 |
| Requirement | 6 | 10 | 15 | 4 | 35 |

(20)

21. A project consists of seven activities for which the relevant data are given below:

| Activity | Preceding Activities | Activity Duration (Days) |
|---|---|---|
| A | _ | 4 |
| B | _ | 7 |
| C | _ | 6 |
| D | A, B | 5 |
| E | A, B | 7 |
| F | C, D, E | 6 |
| G | C, D, E | 5 |

Draw the network. Find the Critical Path and the project completion time. (20)

22. A department has four subordinates and four tasks are to be performed. The subordinates differ in efficiency and the tasks differ in their difficulties. The estimate of time (in man-hours) each man would take to perform each task is given by

| | | Task a | b | c | d |
|---|---|---|---|---|---|
| | 1 | 18 | 26 | 17 | 11 |
| | 2 | 13 | 28 | 14 | 26 |
| Subordinate | 3 | 38 | 19 | 18 | 15 |
| | 4 | 19 | 26 | 24 | 10 |

How should the tasks be allotted to men so as to optimize the total man-hours? (20)

########