LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – APRIL 2010

MT 2961 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Date & Time: 26/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART-A

Answer all the questions : (10x2 = 20)

1) Find the constant C if the following represents the probability mass function of a random variable X .

$$p(x) = C\left(\frac{1}{3}\right)^x, x\ C\left(\frac{1}{3}\right)^x,\ x = 1,2,3\ldots \text{ zero elsewhere .}$$

2) Let the pdf of a continuous type random variable X be
$f(x) = (x+2)/18$, $-2 < x < 4$, zero elsewhere . Find $P(X^2 < 9)$.

3) Define convergence in distribution of a sequence of random variables $\{X_n\}$ to X .

4) Define periodicity of a Markov chain . When do you say that state i is aperiodic?

5) If A and B are independent events . show that A and B^c are independent.

6) State central limit theorem .

7) Obtain the MGF of a r.v with probability mass function
$p(x) = \left(\frac{1}{2}\right)^x$ x = 1,2,3,…. zero elsewhere .

8) Show that communication of states in a Markov chain satisfy Transitive relation .

9) State the postulates of Poisson process .

10) State any two properties of distribution function .

PART-B

Answer any 5 questions : (5 x 8 = 40)

11) State and prove Bayes theorem .

12) Derive the mean and variance of gamma distribution with two parameters .

13) Let X_1 , X_2 ….X_n be n independent observations from $p(x) = p^x q^{1-x}$
x = 0 ,1 zero otherwise , p+q = 1 . Show that $\sum \frac{X_i}{n}$ converges in probability to p .

14) Let X_n be a Markov chain with transition probability matrix P and states 0,1,2

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

and the initial distribution is $P[X_0 = i] = \frac{1}{3}$, i = 0,1,2 .

Find i) $P[X_3 = 2, X_2 = 1, X_1 = 0, X_0 = 0]$
ii) $P[X_2 = 1]$
iii) $P[X_3 = 2, X_2 = 1, / X_1 = 1, X_0 = 0]$ (2+4+2)

15) a) Show that communication of states in a Markov chain is an equivalence relation .
b) Show that if $i \leftrightarrow j$ and if i is recurrent then j is also recurrent.

16) Let X and Y have the joint pdf $f(x, y) = 8xy$, $0<x<y<1$ zero otherwise obtain E[X / Y = y] , Var [X / Y = y] .

17) Given the joint distribution of the random variables X and Y as

(x , y)	(0, 0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
P(x , y)	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

Obtain the correlation coefficient between X and Y .

18) Ten pairs of shoes are in a closet . Four shoes are selected at random .Find the probability that there is atleast one matching pair among the four selected .

PART-C

Answer any two questions : (2 x 20 = 40)

19) a) Let A_n be an increasing sequence of events . Show that $P(\lim A_n) = \lim P(A_n)$. Deduce the result for decreasing events.
b) State and prove Boole's Inequality .
c) Give an example to show that pairwise independence does not imply Independence . (10+5+5)

20) a) Show that almost sure convergence implies convergence in probability . Is the converse true? Justify .
b) Let $X_1, X_2 \ldots X_n$ be a random sample from $f(x) = \frac{1}{\theta}$, $0<x<\theta$, Zero elsewhere . Let Y_n be the n^{th} order statistic . Show that Y_n converges in distribution to a degenerate random variable .
c) State and prove Chebyshev's inequality . (10+5+5)

21) a) State the postulates of a Birth and death process . Obtain the Kolmogorov forward and backward differential equations .
b) obtain the expression for $P_n(t)$ in a Yule process . (12+8)

22) a) Let $X_1, X_2, \ldots X_n$ be independent N(0,1) variables . Obtain the pdf of $Y_1 = X_1 / X_2$.
b) Derive the mean and variance of the random variable X with
$P(X = x) = \binom{x+r-1}{r-1} p^r q^x$ $x = 0,1,2\ldots\ldots$
c) A box contains M white and N-M black balls. A sample of size n is drawn i) with replacement ii) without replacement .
Let X denotes the number of white balls. Obtain the probability distribution of X in both the cases . (7+7+6)

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