LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SIXTH SEMESTER – APRIL 2010

MT 6603/MT 6600 - COMPLEX ANALYSIS

Date & Time: 15/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL questions **(10 x 2 = 20 marks)**

1. Show that w = 1 – 2 + 2z^2 satisfies the Cauchy-Riemann conditions everywhere.
2. Show that the function f(z) = xy + iy is analytic nowhere.
3. Define an integral function and give an example.
4. Evaluate $\int_{|z|=2} \frac{2}{(9-z^2)(z+i)} dz$.
5. Is the function $f(z)=\frac{\sin z}{z^3}$ meromorphic? If so, what are its poles?
6. Find the Laurent series representation of $f(z)=\frac{1}{2(1+z^2)}, 0<|z|<1.$
7. Find the residue of $f(z)=\frac{\sinh z}{z^4}\ at\ z=0.$
8. When do you call a mapping conformal? Is the mapping $f(z)=\bar{z}$ conformal? Justify your answer.
9. State argument principle.
10. Find the coefficient of magnification of $f(z)=z^2\ at\ z=1+i.$

PART – B

Answer any FIVE questions **(5 x 8 = 40 marks)**

11. Prove that the function

$$f(z)=\begin{cases}\frac{z^3}{\bar{z}^2} & if\quad z\neq 0\\ 0 & if\quad z=0\end{cases}$$

is nowhere differentiable.

12. Show that the function u = y^3 – 3x^{2}y is harmonic and find the corresponding analytic function.
13. Find the bilinear transformation which takes the points 1, 0, -1 into i , 1 , ∞.
14. Evaluate $\int_C \frac{\sinh 2z}{z^4} dz$, where C is the boundary of the square whose sides lie along the lines $x=\pm 2$ and $y=\pm 2$, described in the positive sense.

15. State and prove Rouche's theorem.
16. Prove that cross-ratio is invariant under Mobions transformation.
17. State and prove the maximum modulus theorem.
18. State and prove Liouville's theorem.

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. (a) Let f(z) = u (x,4) + iv(x,y) be a function defined on the region D such that u and v and their first order partial derivatives are continuous in D. If the first order partial derivatives of u and v satisfy the C – R. equations at $(x_0, y_0) \in$ D, show that f is differentiable at $z_o = x_o + iy_o$.

 b) Let $\sum_{n=0}^{\infty} a_n z^n$ be a given power series. Show that there exists a number R such that $0 \le R \le \infty$ such that

 (i) the series converges absolutely for every z with $|z| < R$.

 (ii) If 0< ρ < R, the convergence is uniform in $|z| \le \rho$.

 (iii) If $|z| = R$, the series diverges.

20. a) State and prove the Laurent's theorem.

 b) Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in the region 1 < $|z| < 2$.

21. a) State and prove the residue theorem.

 b) Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+\sin\theta)^2}$.

22. For transformation $w = \frac{1}{z},\ z \neq 0,$ prove the following:

 a) $|z| < 1$ is mapped onto $|z| > 1$ and vice versa.

 b) Circles not passing through the origin are mapped onto circle not passing through the origin.

 c) Circles passing through the origin are mapped onto straight lines and vice – versa.

 d) Interior of circles go over to half planes and half planes to circular regions.

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