LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

Date : 03/05/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer all questions. Each question carries 20 marks.

1. (a) If the Wronskian of two functions x_1 and x_2 on the interval I is not zero for atleast one point of I, then prove that the functions x_1 and x_2 are linearly independent on I.

(OR)

(b) Prove that $c_1t + c_2t^2 + c_3t^3$, $t \geq 0$, is a solution of $t^3x'''(t) - 3t^2x''(t) + 6tx'(t) - 6x(t) = 0$. (5)

(c) Prove that $uL(v) - vL(u) = a_0(t)\frac{d}{dt}W[u,v] + a_1(t)W[u,v]$, where u, v are twice differentiable functions and a_0, a_1 are continuous on I. Also deduce Abel's formula.

(OR)

(d) By the method of variation of parameters, find the general solution of $x'''(t) - x'(t) = e^t$. (15)

2. (a) Find the indicial equation of $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (3-x)y = 0$.

(OR)

(b) Find the indicial equation of $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$. (5)

(c) Solve by Frobenius method, $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$.

(OR)

(d) Show that the generating function for the Legendre polynomial is $\frac{1}{\sqrt{1-2tx+t^2}} = \sum_{n=0}^{\infty} t^n P_n(x)$ if $|t| < 1$ and $|x| \leq 1$. (15)

3. (a) State and prove Rodriguez's Formula.

(OR)

(b) For the Bessel function, show that (i) $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$ and (ii) $J_n{}'(x) = J_{n-1}(x) - \frac{n}{x}J_n(x)$ (5)

(c) Solve the Bessel's equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$.

(OR)

(d) State and prove the Integral representation of $2F_1$ (α; β; γ; x). (15)

4. (a) State and prove the sufficient condition for the validity of Lipschitz condition.

(OR)

(b) Let *x* and *y* be the solutions of Sturm-Liouville problem such that $[pW(x,y)]_A^B = 0$, where *W*(*x*, *y*) is the Wronskian of *x* and *y*. Prove that $\int_A^B r(s)\,x(s)y(s)ds = 0$. (5)

(c) State and prove Picard's theorem for boundary value problem.

(OR)

(d) State Green's function. Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0$, $a \le t \le b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$. (15)

5. (a) Define a stable system. Prove that $x' = 0$ is stable.

(OR)

(b) Prove that the null solution of equation $x' = A(t)x$ is stable if and only if a positive constant k exists such that $|\phi(t)| \le k, t \ge t_0$. (5)

(c) Discuss the stability of autonomous systems.

(OR)

(d) By Lyapunov direct method, discuss the stability of $x' = Ax$. (15)

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