LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2013**

MT 5508/MT 5502 - LINEAR ALGEBRA

Date: 13/05/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

Answer ALL questions: (10 x 2 = 20 marks)

1. Show that two elements (a, b) and (c, d) of R^2 are linearly independent if and only if ad – bc = 0.
2. Is the union of subspaces a subspace? Justify.
3. Define homomorphism of a vector space into itself.
4. Define rank and nullity of a vector space homomorphism $T : U \rightarrow V$.
5. Define orthnormal set.
6. Normalize (1+2i, 2-i, 1-i) in C^3 relative to the standard inner product.
7. Show that the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
8. If A and B are Hermitian, show that AB – BA is skew Hermitian.
9. Give a characterization of a Hermitian linear transformation.
10. Show that $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is unitary.

PART – B

Answer any FIVE questions: (5 x 8 = 40 marks)

11. Prove that a non empty subset W of a vector space V over F is a subspace of V if and only if $a w_1 + b w_2 \in W$ for all a, b $\in$ F, $w_1, w_2 \in w$.
12. For a vector space V over F, prove that if U and W are subspaces of V, then $U + W = \{u + w / u \in U,\ w \in W\}$ is a subspace of V.
13. Show that the vectors (1, 1,) and (-3, 2) in R^2 are linearly independent over R, the field of real numbers.
14. If V is a vector space of dimension n and U is a subspace of V, then prove that U has finite dimension and $\dim U \le n$.
15. For any two vectors u, v in V, Prove that $\|u + v\| \le \| u \| + \| v \|$.
16. Prove that $T \in A(v)$ is singular if and only if there exists an element $v \neq 0$ in V such that T (v) = 0.

17. If $T \in A(V)$ and $\lambda \in F$, prove that λ is an eigen value of T if and only if $\lambda I - T$ is singular.

18. Find the rank of the matrix $A = \begin{bmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{bmatrix}$.

PART – C

Answer any TWO questions: (2 x 20 = 40 marks)

19. a) Prove that the vector space V over F is a direct sum of two of its subspace, W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

b) If V is a vector space of finite dimension and w is a subspace of V, then prove that $\dim \frac{V}{W} = \dim V - \dim W$. (10 +10)

20. a) Let $T : U \to V$ be a homomorphism of two vector spaces over F, and suppose that U hasfinite dimension. Then prove that dim U = dim = ker T + dim Im T.

b) Is $T : R^2 \to R$ defined by T (a,b) = ab, a vector space homomorphism? Justify.

(12 + 8)

21. Prove that every finite – dimensional inner product space has an orthonormal set as a basis.

22. a) Prove that $T \in A(V)$ is invertible if and only if T maps V onto V.

b) Show that any square matrix A can be expressed uniquely as the sum of a symmetric and a skew – symmetric matrices. (10 + 10)

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