LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2014**

MT 1902 - MATHEMATICS FOR COMPUTER APPLICATIONS

Date : 07/04/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

Part A (Answer ALL questions) **2 x 10 = 20**

1. Define Lattice.
2. With usual notations prove that (i)$a * a = a$ (ii)$a * b = b * a$.
3. Define context free grammar.
4. What are the logic operators?
5. Let G = (N, T, P, S), where $N = \{S\}, T = \{a\}, P: \{S \rightarrow SS, S \rightarrow a\}$. Check whether G is ambiguous or unambiguous.
6. Give a deterministic finite automata accepting the set of all strings over {0, 1} containing 3 consecutive 0's.
7. State the Pigeon hole principle.
8. Define a bipartite graph with an example.
9. Prove that every cyclic group is abelian.
10. Define ring with an example.

Part B (Answer ALL questions) **5 x 8 = 40**

11. (a) Prove that a bijective map of a lattice L into a lattice L' is a lattice isomorphism if and only if its inverse is order preserving.

(or)

(b) Prove that the complement a' of any element 'a' of a Boolean algebra is uniquely determined. Prove also that the map $a \rightarrow a'$ is an anti – automorphism of period $\leq$ 2 and $a \rightarrow a'$ satisfies $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$, $a'' = a$.

12. (a) Write a short note on principal conjunctive normal form and construct an equivalent formula for $\urcorner(P \vee Q) \rightleftarrows (P \wedge Q)$.

(or)

(b) For a grammar $G = (\{S, Z, A, B\}, \{a, b\}, P, S)$ where P consists of the following production:

1. $S \rightarrow aSA$ 4. $Z \rightarrow bB$ 7. $bB \rightarrow bb$
2. $S \rightarrow aZA$ 5. $BA \rightarrow AB$ 8. $bA \rightarrow ba$
3. $Z \rightarrow bZB$ 6. $AB \rightarrow Ab$ 9. $aA \rightarrow aa$

Then show that $L(G) = \{a^n b^m a^n b^m / n, m \geq 1\}$.

13. (a) Let L be a set accepted by a nondeterministic finite automaton. Then prove that there exists a deterministic finite automaton that accepts L.

(or)

(b) Define a Regular Expression.
(c) Explain the equivalence of deterministic finite automata and regular expressions.

14. (a) Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.
(b) Prove that
(i) $A - B = A \cap B'$. (ii) $(A \cup B) \cup C = A \cup (B \cup C)$.

(or)

(c) If G is a group then show that

(i) for every $a \in G, \left(a^{-1}\right)^{-1} = a$

and (ii) for all $a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$.

(d) Prove that $(ab)^2 = a^2b^2$ for all a, b in a group G if and only if G is abelian.

15. (a) Prove that a subgroup N of a group G is a normal subgroup of G iff the product of two left cosets of N in G is again a left coset N in G.

(or)

(b) Prove that the following statements are equivalent for a connected graph G.

(i) G is Eulerian

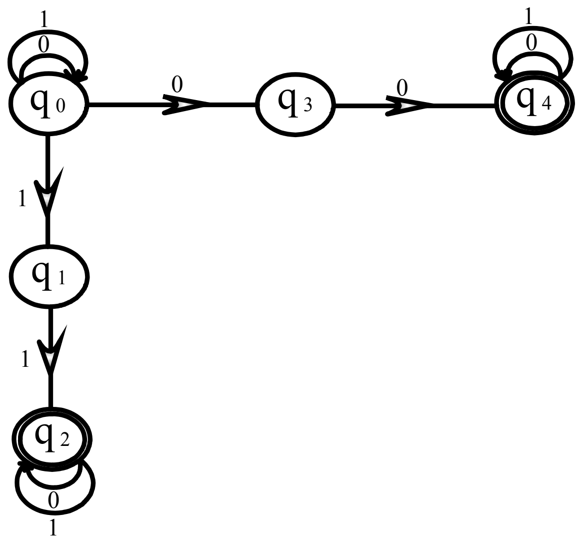
(ii) Every point of G has even degree

(iii) The set of edges of G can be partitioned into cycles.

Part C (Answer ANY TWO questions) **2 x 20 = 40**

16. (a) Define a Non – Deterministic Finite automata.

(b) For the non deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$,



give the transition table and show that 0100110 is in L (M).

(c) Let r be a regular expression. Then prove that there exists an NFA with $\in$-transitions that accepts L (r). **(2+8+10)**

17. (a) Prove that every chain is a distributive lattice.

(b) State and prove pumping lemma.

(c) Prove that if $f : A \rightarrow B$ and $g : B \rightarrow C$ be one-to-one onto functions, then gof is also one-to-one onto and $(gof)^{-1} = f^{-1}og^{-1}$. **(4+10+6)**

18. (a) Let G be a (p, q)graph. Then prove that the following statements are equivalent

(i) G is a tree.

(ii) Every two points of G are joined by a unique path.

(iii) G is connected and p = q + 1.

(iv) G is acyclic and p = q + 1.

(b) State and prove Lagrange theorem.