LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **APRIL 2019**

16UMT5MC01– REAL ANALYSIS

Date: 15-04-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Part-A

Answer ALL the questions. **(10X 2=20)**

1. Define ordered field.
2. Prove that N is unbounded above.
3. Define Metric Space.
4. Define Open Cover.
5. Define complete metric space.
6. Define a bounded set.
7. If f is differentiable at c then prove that f is continuous at c.
8. State Intermediate value theorem for derivatives.
9. Define Monotonic functions.
10. Define Bounded Variation.

PART-B

Answer any FIVE questions. **(5X 8=40)**

11. If $e = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots$, then prove that *e* is irrational.
12. Prove that every subset of a countable set is countable.
13. Let M=R^n. $If\ x = (x_1, x_2, ..x_n)\ and\ y = (y_1, y_2, ...y_n)$ are points in R^n, define

    $d(x,y) = [\sum_{k=1}^{n} (x_k - y_k)^2]^{\frac{1}{2}}$. Show that (M, d) is a metric space.
14. State and prove Heine Borel theorem.
15. Prove that every convergent sequence is Cauchy sequence.
16. Prove that every subset of a compact set is compact.
17. State and prove Rolle's theorem.
18. Prove that if *f* is monotonic on [a, b], then the set of discontinuities of *f* is countable.

PART-C

Answer any TWO questions. **(2 X20=40)**

19. (a) Prove that the set R is uncountable.

(b) State and prove Cauchy- Schwarz inequality. **(10 +10)**

20.(a) Let (X, d) be a Metric space, then prove that the following

(i) The intersection of an arbitrary collection of closed sets in X is closed in X

(ii) The intersection of a finite collection of open sets in X is open in X.

(b) State and prove Bolzano-Weierstrass theorem for R. **(10 +10)**

21. (a) Let f and g be real valued functions defined on a metric space (X,d). If f and g are continuous at $x_0 \in X$, then prove that (i) kf (ii) f+g and (iii) fg are continuous at $x_0 \in X$.

(b) State and prove Generalized mean value theorem. **(10 +10)**

22. (a) State and prove Taylor's theorem.

(b) Let f be functions of bounded variation defined on [a, b] and c ∈ (a, b). Then prove that f is bounded variation on [a, b] as well as on [a, c] and $V_f[a,b] = V_f[a,c] + V_f[c,b]$.

(10 +10)

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