LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **APRIL 2019**

16UMT5MC03– LINEAR ALGEBRA

Date: 22-04-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Part-A

Answer ALL the questions. **(10X 2=20)**

1. Define a vector Space.

2.If V is a vector space over F, then prove that $-(\alpha v) = (-\alpha)v$ for $\alpha \in F$ and $v \in V$.

3. Define a inner product space.

4. Prove that $W^{\perp}$ is a subspace of V.

5. Define regular linear transformation.

6. Prove that if V is finite dimensional over F and if $T \in A(V)$ is right invertible then it is invertible.

7.When the linear transformation $S, T \in A(V)$are said to be similar?

8. Define matrix of a linear transformation T.

9. $T \in A(V)$is unitary if and only if $TT^* = 1$

10.If N is normal and $AN = NA$ prove that $AN^* = N^*A$

PART-B

Answer any FIVE questions. **(5X 8=40)**

11.Prove that if V is the internal direct sum of $U_1, U_2, \dots U_n$, then V is isomorphic to the external direct sum of $U_1, U_2, \dots U_n$.

12.State and prove Schwarz inequality.

13.If V is finite dimensional over F, then $T \in A(V)$is invertible if and only if the constant term of the minimal polynomial for T is not zero.

14.Prove that if V is ndimensional over F and if $T \in A(V)$ has the matrix$m_1(T)$ in the basis$v_1, v_2, v_3, \dots . v_n$and the matrix $m_2(T)$in the basis$w_1, w_2, w_3, \dots . w_n$ of V over F, then there is an element $c \in F_n$such that $m_2(T) = cm_1(T)c^{-1}$

15. If $T \in A(V)$ is such that $< vT, v > = 0$ for all $v \in V$ then show that $T = 0$.

16. If $T \in A(V)$ is Hermitian, then all its characteristic roots are real.

17.If $S, T \in A(V)$and if S is regular, then T *and* STS^{-1} have the same minimal polynomial.

18. If V is a vector space over F and if W is a subspace of V, then prove that V/W is a vector space over F.

PART-C

Answer any TWO questions. **(2 X20=40)**

19 (a)Prove that if $v_1, v_2, \dots v_n$ is basis of V over F and if $w_1, w_2, \dots w_m$ in V are linearly independent over F, then $m \leq n$.

(b)If V is a vector space over F and if W is a subspace of V. Prove that W is finite dimensional,dim $W \leq dimV$ *and* $dim\, V/W = \dim V - dimW$. **(10+10)**

20 (a) State and prove Gram-Schmidt orthogonalization process.

(b)If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then for any polynomial $q(x) \in F[x], q(\lambda)$ is a characteristic root of $q(T)$. **(12+8)**

21 (a)If V is finite dimensional over F then for $S, T \in A(V)$, prove the following

(i)$r(ST) \leq r(T)$

(ii) $r(TS) \leq r(T)$

(iii)$r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

(b) If $T \in A(V)$has all its characteristic roots in F, and then prove that there is a basis of V in which the matrix of T is triangular. **(10+10)**

22.If $T \in A(V)$ *then* prove that the following

(i)$T^* \in A(V)$ (ii) $(S + T)^* = S^* + T^*$ (iii) $(\lambda S)^* = \bar{\lambda} S^*$

(iv) $(ST)^* = T^* S^*$ for all $S, T \in A(V)$.

******