



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – MATHEMATICS

SIXTH SEMESTER – APRIL 2019

16UMT6MC01- COMPLEX ANALYSIS

Date: 04-04-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer **ALL** questions.

(10X2=20)

1. Show that $u(x, y) = 2x(1 - y)$ is harmonic.
2. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.
3. Evaluate $\int_C \frac{dz}{z-3}$ where C is the circle $|z - 2| = 5$.
4. Define simply connected region.
5. Write the Maclaurin series expansion of the function $f(z) = e^z$.
6. State Laurentz's theorem.
7. Determine and classify the singular points of $f(z) = \sin\left(\frac{1}{z}\right)$.
8. Find the residue of $\cot z$ at $z = 0$.
9. Find the fixed point of a Bilinear transformation $w = \frac{z-1}{z+1}$.
10. Define conformal mapping.

PART-B

Answer any **FIVE** questions

(5x8=40)

11. Prove that an analytic function with constant modulus is constant.
12. State and prove Cauchy-Integral formula.
13. State and prove fundamental theorem of algebra.
14. State and prove Cauchy Residue theorem.
15. Find the Bilinear transformation which maps the points $z_1 = 0, z_2 = -i, z_3 = -1$ into the points $w_1 = i, w_2 = 1, w_3 = 0$.
16. State and prove maximum modulus theorem.
17. Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$ using contour integration.
18. Discuss the transformation $w = \frac{1}{z}$.

PART-C

Answer any **TWO** questions.

(2X20=40)

19. (a) Derive Cauchy-Riemann equation in polar form.
(b) Prove that the function $u = 2x - x^3 + 3xy^2$ is harmonic. Also find the function $f(z) = u(x, y) + iv(x, y)$ such that $f(z)$ is analytic. **(10+10)**
20. (a) State and prove Cauchy-Goursat theorem.
(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$. **(12+8)**
21. (a) State and prove Taylor's theorem.
(b) State and prove Liouville's theorem. **(12+8)**
22. (a) State and prove Rouché's theorem.
(b) State and prove Argument principle. **(10+10)**

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