LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18PMT1MC05 – PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 03-11-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer **ALL** questions:

1. (a)With usual notations prove that $-1 \leq \gamma_{XY} \leq 1$. **(5)**

(OR)

(b) A random variable has the following probability function:

Values of X, x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6)$(iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a. **(5)**

(c) For the joint probability distribution of two random variables X and Y given below:

Y	1	2	3	Total
X	4			
1	4/36	3/36	2/36	10/36
2	1/36			9/36
3	1/36	3/36	3/36	8/36
4	2/36			9/36
	5/36	1/36	1/36	
	1/36			
	1/36	2/36	1/36	
	5/36			
Total	11/36	9/36	7/36	1
	9/36			

Find (i) the marginal distributions of X and Y.

(ii) Conditional distribution of X given the value of $Y=1$ and that of Y given the value of $X=2$.

(15)

(OR)

(d) Let (X,Y) be a two dimensional random variable uniformly distributed over the triangular region bounded by $y=0, x=3$ and $y=\frac{4}{3}x$. Obtain the correlation coefficient between X and Y. **(15)**

2. (a) State and prove Chebychev's theorem. **(5)**

(OR)

(b) For the geometric distribution $p(x)=2^{-x}; x=1,2,3,\ldots$, prove that Chebychev's inequality gives $P\{|X-2| \leq 2\} > \frac{1}{2}$, while the actual probability is $\frac{15}{16}$. **(5)**

(c) State and prove two Borel-Cantelli Lemmas. **(15)**

(OR)

(d) State and prove De-Moivre's Laplace theorem. **(15)**

3. (a)Prove that the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$, for a sample of unit size is $2x$, x being the sample value. Show also that the estimate is biased. **(5)**

(OR)

(b) Let $x_1, x_2, \dots, x_n$ be a random sample from $N(\mu, \sigma^2)$ populations. Find sufficient estimators for μ and σ^2. **(5)**

(c) If T_1 and T_2 are M. V. U. estimators for $\gamma(\theta)$, then prove that $T_1 = T_2$, almost surely. **(15)**

(OR)

(d) Let T_1 and T_2 be unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively and $\rho = \rho_0$ be the correlation between them. Then prove that $\sqrt{e_1 e_2} - \sqrt{(1-e_1)(1-e_2)} \le \rho \le \sqrt{e_1 e_2} + \sqrt{(1-e_1)(1-e_2)}$. **(15)**

4. (a) Write any three advantages and drawbacks of Non-Parametric Methods over parametric methods. **(5)**

(OR)

(b) If $x \ge 1$ is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population, $f(x, \theta) = \theta \exp(-\theta x), 0 \le x < \infty$, obtain the values of type I and type II errors. **(5)**

(c) (i) Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.

(ii) Write down the procedure for Sign test. **(9+6)**

(OR)

(d) State and prove Neyman-Pearson Lemma. **(15)**

5. (a) Explain the four different classes of stochastic processes. **(5)**

(OR)

(b) Write short notes on Marko Process and Stationary Process. **(5)**

(c) (i) Explain Birth and Death processes.

(ii) If the initial vector $\boldsymbol{P}^{(0)}$ is given, then prove that the n –step transition probabilities are $\boldsymbol{P}^{(n)} = \boldsymbol{P}^{(0)}\boldsymbol{P}^{\boldsymbol{n}}$, $n = 1, 2, \dots$. **(5+10)**

(OR)

(d) State and prove Chapman-Kolmogorov relation in Markov chain. **(15)**

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