



Date: 01-11-2018

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

Part A

Answer ALL Questions.

(10 x 2 = 20)

1. Find the n^{th} derivative of e^{ax} .
2. Prove that the polar subnormal of the curve $r = a\theta$ is constant.
3. When do you say that a series is convergent?
4. Write down the expansions of $\log(1 + x)$ and $\log(1 - x)$.
5. Find $L(t^2 + 2t + 3)$.
6. Find $L^{-1}\left(\frac{s}{s^2+a^2}\right)$.
7. Define symmetric and skew-symmetric matrices.
8. State Cayley-Hamilton theorem.
9. Define binomial distribution.
10. Write down the formula for rank correlation with non-repeated ranks.

Part B

Answer Any FIVE Questions.

(5 x 8 = 40)

11. Find y_n where $y = \frac{3}{(x+1)(2x-1)}$.
12. Sum the series to infinity $\frac{15}{16} + \frac{15 \cdot 21}{16 \cdot 24} + \frac{15 \cdot 21 \cdot 27}{16 \cdot 24 \cdot 32} + \dots$.
13. (i) Find $L(te^{-t} \sin t)$, (ii) Evaluate $\int_0^{\infty} e^{-2t} \sin 3t dt$.
14. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.
15. The average salary of male employees in a firm was Rs. 5,200 and that of females was Rs. 4,200. The mean salary of all the employees was Rs. 5,000. Find the percentage of male and female employees.
16. Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$.
17. Examine the convergence of the series $\sum_{n=0}^{\infty} \frac{n^3+1}{2^{n+1}}$.

18. Solve the system of the following equations using Cramer's rule:

$$2x - y + 3z = 9; \quad x + y + z = 6; \quad x - y + z = 2$$

Part C

Answer Any TWO Questions.

(2 x 20 = 40)

19. (a) If $y = \sin(m\sin^{-1}x)$ prove that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ and

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

(b) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$.

20. (a) Find $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$.

(b) Using Laplace transform, solve $y'' + 2y' + 5y = 4e^{-t}$ given that $y(0) = 0, y'(0) = 0$.

21. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

(b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.

22. (a) Calculate the correlation coefficient for the following data.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(b) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

Age in years	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
No. of members	3	61	132	153	140	51	2
