

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – NOVEMBER 2018

MT 3100 – ALLIED MATHEMATICS FOR PHYSICS

Date: 03-11-2018

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

SECTION-A

(10 x 2 = 20)

Answer ALL the questions

1. Find the n^{th} derivative of e^{2x} .
2. Prove that the sub tangent to the curve $y = a^x$ is of constant length.
3. What is $L[\cos 4t - e^{2t}]$?
4. Find $L^{-1}\left[\frac{2}{(s+4)^2}\right]$.
5. What is the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?
6. Write down the expansion of $\cos \theta$ in ascending powers of θ .
7. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$.
8. Write down the expansion of $\log_e (1-x)$.
9. Write the formula of Poisson distribution.
10. Write down the probability mass function of binomial distribution.

SECTION-B

(5 x 8=40)

Answer any FIVE questions

11. Find the angle of intersection of cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
12. Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -1 & 2 & -1 \end{pmatrix}$.
13. Find the sum to infinity of the series $1 + \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \dots$
14. Prove that any group is isomorphic to a group of permutations.
15. Evaluate $\int_0^{\infty} e^{-2t} \sin 3t dt$ by using Laplace transform.
16. Evaluate $L^{-1}\left[\frac{s}{(s+3)^2+4}\right]$.
17. If $y = \sin^{-1}x$, then prove that $(1 - x^2)y_2 - xy_1 = 0$ and hence show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.
18. Write down the expansion of $\cos 6\theta$ in terms of $\cos \theta$.

SECTION-C

(2 x 20=40)

Answer Any TWO questions

19. a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.
- b) Show that $1 + \frac{2^4}{2!} + \frac{3^4}{3!} + \frac{4^4}{4!} + \dots = 15e$. (12+8)
20. a) Find the minimum value of the function $f(x) = \frac{\log x}{x}$, for $x > 0$.
- b) Find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + e \cos \theta$. (10+10)
21. a) Expand $\sin^3 \theta \cos^5 \theta$ in a series of sines of multiples of θ .
- b) Prove that $\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$. (10+10)
22. Find the mean and standard deviation for the following table giving the age distribution of 542 students.

| Age (in years) | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| No. of students | 3 | 61 | 132 | 153 | 140 | 51 | 2 |
